Circle of Fourths: an orchestral overture

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How do you resolve the circle of fourths? The question has been answered acoustically, arithmetically, philosophically, and pragmatically. But what if you treat the circle as a suspension: how do you resolve that, musically?

Nearly every musician spends some time, first in wonder then in worry, on discovering the circle of fourths (or fifths). Play four ascending notes in a major scale, say C-F. Then another: F-Bb. Continuing thus, and setting Gb = F#, we have C-F-Bb-Eb-Ab-Db-Gb(=F#)-B-E-A-D-G-C. On a piano, Gb *does* equal F# so, after 12 fourths, we have covered five octaves and come back 'home' to C — hence the circle. There's the wonder.

The worry arises because a perfect fourth has a frequency ratio of 4/3 and an octave has 2/1. So 5 octaves = $2^5 = 32$, while $(4/3)^{12} = 31.6$. Out by 1.3% or a quarter of a semitone: ouch! This interval is called the Pythagorean comma [1], which shows that it's not a new problem! Minimising the potential mistuning caused by this and related problems is called temperament [2]. There are many temperaments, all with disadvantages: Lakes of ink have been poured into the problem of temperament over more than two millennia. Equal temperament is one approximation: make all the fourths and all the semitones equal. This gives each semitone a ratio of $2^{1/12}$ or 1.059, and fourth a ratio of 1.335. This is pretty close to 4/3 and causes no distress. The equal tempered third, at $2^{1/3}$ = 1.260 instead of 5/4, is often unacceptable, say on two flutes or violins, where it produces a Tartini tone that clashes unpleasantly with the bass notes in the chord. An explanation using sound files and animations is given by the present author [3].

Equal temperament is used on many, but not all, keyboard instruments. Pianos usually use equal temperament: on this instrument, the strings in a triplet are tuned to slightly different pitches and the transients are strong. Together, these disguise the beatings from equal tempered thirds. (Music historians generally believe that Bach wrote The Well-Tempered Clavier to illustrate one of the well temperaments, not equal temperament.)

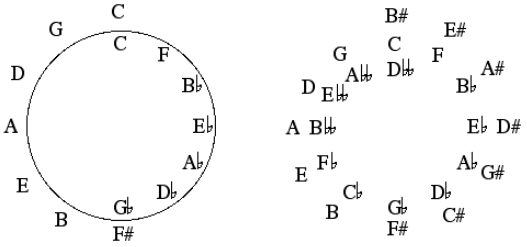

Figure 1: In equal temperament, as used on many keyboards, the circle is forced to close, as illustrated at left. If pure fourths are retained, it is sometimes represented as a spiral.

Quite apart from the historical, physical and philosophical interest in the circle, I was attracted by the melodic and harmonic possibilities of successive fourths. Stacked up over five octaves, the circle is an interesting chord and building it from the bass creates musical tension. By keeping the pedal notes loud and distributing it through the orchestra, any performance is unlikely to produce serious temperamental problems. There is, however, the musically interesting question raised above? Where to go after a chord that contains 12 different notes: how can it be resolved? This work has a few different answers.

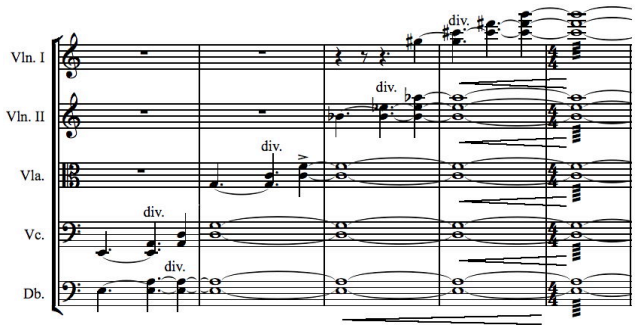

Figure 2: This section from the string parts shows an almost complete circle: in the full score, it is closed by E7 on the piccolo. The complete score [3] shows how the circle is distributed in the orchestra (bars 11, 43 and 152).

The piece is mainly in 8:8 time (3+3+2 quavers) with, for contrast, some interjections of 3+3+3+2+2, a 3:4 slow section in the middle and some syncopated four at the end. One of its themes uses two rising fourths as a nucleus, the others use a variable number of descending scale steps.

Why a circle of fourths, rather than fifths? First, a circle of fifths is seven octaves, which stretches the orchestral range and would make it difficult to balance chords. Second, when musicians depart from perfect tuning, they usually stretch intervals. Stretching a circle of fourths reduces the temperament problem, stretching fifths increases it. Third, and most importantly, I preferred melodic and harmonic possibilities of the fourths.

This overture was commissioned by the UNSW Orchestra to celebrate its 100th concert in 2010. It has since been performed in the Sydney Symphony's Playerlink program. The score and parts are available for download [4]. I ask that orchestras performing the work make a small donation, comparable with the cost of hiring an orchestral set, to a suitable charity.

References

[1] http://en.wikipedia.org/wiki/Pythagorean_comma
[2] http://en.wikipedia.org/wiki/Musical_temperament
[3] http://www.animations.physics.unsw.edu.au/waves-sound/interference/
[4] http://www.phys.unsw.edu.au/~jw/music/circle/