# Single and multiple scattering contributions to circumsolar radiation

M. A. Box and A. Deepak

Single and multiple scattering contributions to the circumsolar radiation along the almucantar and sun vertical have been computed by a Gauss-Seidel solution to the radiative transfer equation. In the near forward direction, the multiple scattering contributions are significant for optical depths of the order of 0.4. However, the shape of the angular distribution of almucantar radiance up to 10° appears less sensitive to multiple scattering. The results have been compared against an existing radiative transfer code.

## Introduction

In recent years, measurements of the solar aureole almucantar radiance have been successfully used to determine the size distribution (SD) of atmospheric aerosols.[1–4] Solar aureole is the region of enhanced brightness within about 20° around the sun's disk, predominantly due to aerosol scattering; the almucantar is a scan of constant zenith angle through the sun. In all these cases, one of the main assumptions made in the retrieval of aerosol SD has been that of single scattering (SS). Questions have been raised as to whether multiple scattering (MS) contributions to the solar aureole have any significant effect on the retrieval of the SD. This question will be answered in two steps. The first step is to compare the contributions to the angular distribution of the almucantar radiance in the forward direction due to MS relative to those due to SS. The second step is to determine the effect of MS on the retrieval of the aerosol SD. In this paper, only the first step will be considered, the second being left to a subsequent publication. Thus, for the purpose of calculating the SS and MS contributions, a computer code was developed which employed the Gauss-Seidel iterative approach to the solution of the radiative transfer equation for a plane parallel atmosphere composed of air molecules, ozone, and aerosol particles. Our code is essentially similar to the radiative transfer (RT) code written by Dave[5] except in the construction of the source matrix. The difference in the two codes is outlined below. The latter code, referred to as Dave's RT code, has been widely used by different researchers[6–8] in recent years.

Prior to utilizing our code in actual problems, we decided to check some results obtained by it for SS and MS radiance contributions in an aerosol atmosphere. But, unfortunately, we were unsuccessful in locating a set of standard tables of downwelling and upwelling radiance and polarization for atmospheres containing inhomogeneously distributed aerosol particles. Dave and Furukawa's tables[9] and Coulson *et al.*'s tables[10] are meant for only molecular atmospheres, with and without ozone, respectively. Therefore, the following strategy was adopted. First, it was decided to check the results obtained by ours and Dave's RT codes, using identical molecular data input, against the tables in Refs. 9 and 10. If the three sets of values agreed to within say 1–2%, one could assume the two codes work correctly for both SS and RT in molecular atmosphere. The next step was to add aerosols into the molecular atmosphere and use identical aerosol data input for the two codes and compare their results. If the two sets of results agreed with one another within 1%, it would be safe to assume that our code was as accurate as Dave's RT code, which we treated as our standard against which to check. (Plans for checking our code against other codes were considered but abandoned in view of the high computer costs involved.)

## Gauss-Seidel Technique

Before discussing the results of the aforementioned computations, a brief description of the two codes and their similarities and differences seems appropriate. Both codes are based on a Fourier decomposition method of solving the RT equation for downwelling and upwelling radiances and polarization. Following Herman's method,[11] the atmosphere is divided into a

The authors are with Institute for Atmospheric Optics & Remote Sensing, P.O. Box P, Hampton, Virginia 23666.
Received 19 April 1978.
0003-6935/78/1201-3794$0.50/0.

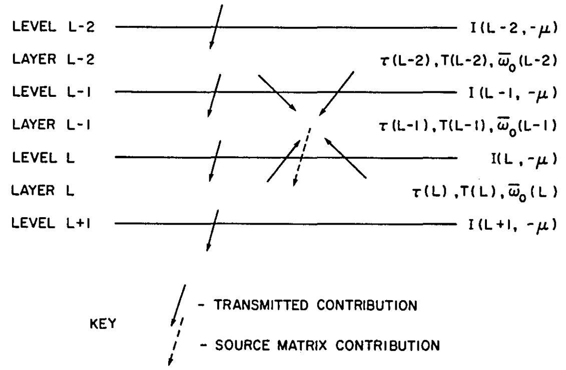


Fig. 1. Almucantar radiance vs azimuth angle (0–180°) for $I_{SS}$, $I_{RT}$ (Dave), and $I_{RT}$ (Our).

**Table I. Input Data**

| | |
|---|---|
| Wavelength | $\lambda = 0.55\ \mu m$ |
| Rayleigh optical depth | $\tau_R = 0.1$ |
| Particulate scattering optical depth | $\tau_{PS} = 0.01026$ |
| Particulate absorption optical depth | $\tau_{PA} = 0.00236$ |
| Ozone absorption optical depth | $\tau_{O_3} = 0.0$ |
| Total optical depth | $\tau = 0.11262$ |
| Solar zenith angle | $\theta_{SUN} = 60°$ |
| Unattenuated solar flux | $F = 100\pi$ |
| *Aerosol Characteristics* | |
| Refractive index | $m = 1.50 - i0.03$ |
| Size distribution | $n(r) = C,\ 0.03\ \mu m \le r \le 0.1\ \mu m$ |
| | $= C(r/0.1)^{-4},\ 0.1\ \mu m \le r \le 2.0\ \mu m$ |
| | $= 0$, otherwise |

The value of $C$ is determined by $\tau_{PS}$.
Volume scattering coefficient $\beta_{scat} = 4.85 \times 10^{-10}\ cm^{-1}$ per particle
Volume absorption coefficient $\beta_{abs} = 1.12 \times 10^{-10}\ cm^{-1}$ per particle
Volume extinction coefficient $\beta_{ext} = 5.97 \times 10^{-10}\ cm^{-1}$ per particle

**Table II. Almucantar Radiances $I_{SS}$, $I_{RT}$ (Dave), and $I_{RS}$ (Our) Obtained by SS Theory and Dave's and Our Codes, Respectively**

| Azimuth angle (deg) | $I_{RT}$ (Dave) $A = 0.0$ | $I_{RT}$ (Dave) $A = 0.25$ | $I_{SS}$ ($A = 0.0$) | $I_{RT}$ (Our) $A = 0.0$ | $I_{RT}$ (Our) $A = 0.25$ |
|---|---|---|---|---|---|
| 0 | 15.556 | 16.627 | 17.015 | 18.003 | 19.056 |
| 30 | 7.864 | 8.935 | 7.209 | 8.142 | 9.195 |
| 60 | 5.450 | 6.522 | 4.655 | 5.452 | 6.506 |
| 90 | 4.086 | 5.158 | 3.352 | 4.039 | 5.092 |
| 120 | 3.843 | 4.915 | 3.123 | 3.780 | 4.833 |
| 150 | 4.293 | 5.365 | 3.530 | 4.214 | 5.268 |
| 180 | 4.588 | 5.660 | 3.796 | 4.501 | 5.550 |

number of levels, and a Gauss-Seidel iterative scheme is employed, passing first in a downward direction, then upward, repeating the procedure until convergence is reached. In what follows we consider a downward pass, the extension to upward pass being similar.

The downward intensity at level $L$ (Fig. 1) is taken to be the downward intensity at level $L - 1$, attenuated by its passage through the intervening layer, plus the source matrix contribution from this layer. A layer is characterized by an optical thickness $\tau$, single-scattering albedo $\bar{\omega}_0$, and turbidity factor $T$ (= the ratio of particulate to total extinction coefficients), each assumed constant.

To obtain the source matrix for a given layer, one combines the optical properties of the layer with the average of the intensities of the levels which bound it. Using the Gauss-Seidel technique, the most recent values of the intensity are used at each iteration stage. Dave's code requires four subroutine calls, one each for the upward and downward intensities of each level involved, whereas, our code requires only two calls, the two levels being handled simultaneously, which results in a saving of about 20% in execution time.

## Comparison of the Computer Codes

To illustrate the difference between codes, we have used the data set which Dave employed as his example in his documentation.[5] The important input data information is listed in Table I. The vertical aerosol profile is that of Ref. 5, and the ozone absorption is ignored. The output from Dave's code checks out to be in complete agreement with that published in Ref. 5. However, the output from our code differs from these results, most notably in the near forward direction. This is illustrated in Tables II and III and in Fig. 2.

**Table III. Sun-Vertical Radiances $I_{SS}$, $I_{RT}$ (Dave), and $I_{RT}$ (Our) Obtained by SS Theory and Dave's and Our RT Codes, Respectively**

| Zenith angle (deg) | $I_{RT}$ (Dave) $A = 0.0$ | $I_{RT}$ (Dave) $A = 0.25$ | $I_{SS}$ ($A = 0.0$) | $I_{RT}$ (Our) $A = 0.0$ | $I_{RT}$ (Our) $A = 0.25$ |
|---|---|---|---|---|---|
| 0 | 2.486 | 3.044 | 2.157 | 2.471 | 3.017 |
| 10 | 2.903 | 3.470 | 2.553 | 2.905 | 3.459 |
| 20 | 3.514 | 4.107 | 3.141 | 3.553 | 4.132 |
| 30 | 4.407 | 5.048 | 4.030 | 4.526 | 5.152 |
| 40 | 5.812 | 6.532 | 5.500 | 6.112 | 6.816 |
| 50 | 8.574 | 9.423 | 8.613 | 9.388 | 10.220 |
| 60 | 15.556 | 16.627 | 17.015 | 18.003 | 19.056 |
| 70 | 15.225 | 16.730 | 15.283 | 16.624 | 18.110 |
| 80 | 20.889 | 23.495 | 19.720 | 21.892 | 24.482 |

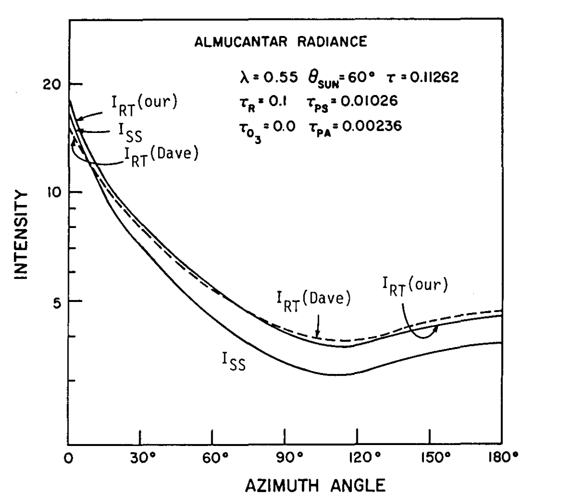


Fig. 2. Schematic illustration of atmospheric layers, transmitted and source matrix contribution to sky radiance.

Rather than reproduce the entire output of each code (a total of more than 3000 numbers each), we give the results for the almucantar scan (observation zenith angle equal to solar zenith angle) in Table II, and, the sun-vertical scan (observation azimuth angle equal to solar azimuth angle) in Table III. In addition, the almucantar results for no ground reflectivity ($A = 0$) are plotted in Fig. 2. Each table consists of five columns: the first two give Dave's results for ground reflectivities ($A$) of 0.0 and 0.25; the last two give results of our code; the middle column gives the single scattering (SS) results.

The most striking (and significant) result is contained in the first row of Table II, where we see that the SS contribution for scattering angle $\psi = 0°$ is greater than Dave's RT results for both values of $A$. This can also be seen clearly in Fig. 2, where Dave's results are shown as a broken line. Turning to Table III, we see that the SS contribution is higher than Dave's result with $A = 0.0$ for zenith angles of 50°, 60°, and 70°.

In order to make sure that our SS results are indeed correct, we compared them with Dave and Furukawa's tables[9] for air, ozone, but no aerosols and found agreement between them to within 0.5% for zenith angles up to 75°. For zenith angles of 80° and 85°, the discrepancy was only slightly greater. We feel the reason for this small discrepancy is due to the Fourier decomposition of the phase function used in our (and Dave's) code, which clearly must be truncated at some finite order.

As an alternative verification, we have used the formula of Green *et al.*[1] for the almucantar SS radiance, viz.,

$$I_{SS}(\psi) = F \sec\theta \exp(-\tau \sec\theta)[\tau_R R(\psi) + \tau_{PS} P(\psi)]/4\pi, \quad (1)$$

where $R$ and $P$ are the (normalized) Rayleigh and particulate phase functions, $\psi$ is the scattering angle, and $\theta$ is the solar zenith angle. Equation (1) is a special case of the more general formula for SS scattering,[1] since for almucantar scan the observation angle is equal to $\theta$. To obtain the first value in Table II, we choose $\psi = 0°$, so that $R(0) = 3/2$. From earlier results in Dave's documentation,[5] $P(0) = 26.94$. Using these values along with the data in Table I yields $I_{SS}(0) = 17.02$, in excellent agreement with our result in Table II.

Figure 2 shows that Dave's code gives a flatter almucantar radiance scan than our code, implying a less forward-peaked phase function. The phase function is constructed as a linear combination of Rayleigh and particulate phase functions, and these results suggest that Dave's code incorporates too little of the particulate contribution to the total phase function.

In constructing the source matrix for, say, layer $L$, it is necessary to combine the scattering properties of layer $L$ with the intensities of levels $L + 1$ and $L$. It seems that in Dave's code, the intensities of level $L + 1$ are combined with the appropriate scattering properties of layer $L$, but the intensities of level $L$ are combined with the turbidity factor of layer $L - 1$ (i.e., the layer above the correct layer). As the scale height of the aerosol profile is much smaller than that for the molecular profile (in the troposphere), the turbidity factor decreases with height. Thus, by taking the turbidity factor for the layer above the required layer, Dave's code appears to include insufficient aerosol scattering.

Thus we see from Tables II and III that, whereas the results for SS and total radiance ($I_{SS}$ and $I_{RT}$, respectively) obtained by the two codes (i.e., ours and Dave's) for the molecular atmosphere agree to within 1–2%, those for molecular plus aerosol atmosphere show a discrepancy. However, for the latter case, the values of $I_{RT}$ obtained by Dave's code turn out to be less than $I_{SS}$ values in the forward direction, which is obviously incorrect, whereas $I_{RT}$ values obtained by our code are greater than the $I_{SS}$ values for all angles, as they should be. It has been suggested to us that the discrepancy in the former case might disappear if $\theta$ integration is performed over smaller angle intervals (e.g., 2°, instead of the 10° in this case) and the number of layers is in-

**Table IV. Normalized Almucantar Radiance Values $I^N_{SS}$ and $I^N_{RT}$ and for Different $\theta_{sun}$, $\tau_R$, and $\tau_{PS}$**

| Azimuth angle (deg) | Scattering angle (deg) | $I^N_{SS}$ | $I^N_{RT}$ | | $(I^N_{RT} - I^N_{SS})/I^N_{SS}$ | |
|---|---|---|---|---|---|---|
| | | | $A = 0$ | $A = 0.25$ | $A = 0$ | $A = 0.25$ |
| Solar zenith angle = 30°, $\tau_R = 0.1, \tau_{PS} = 0.2$ | | | | | | |
| Normalization constant = | | 1.0920 | 1.1406 | 1.1574 | 4.5 | 6.0 |
| 0 | 0.0 | 1.0000 | 1.0000 | 1.0000 | 0.0 | 0.0 |
| 4 | 2.0 | 0.9779 | 0.9787 | 0.9790 | 0.1 | 0.1 |
| 8 | 4.0 | 0.9161 | 0.9191 | 0.9202 | 0.3 | 0.4 |
| 12 | 6.0 | 0.8257 | 0.8318 | 0.8342 | 0.7 | 1.0 |
| 16 | 8.0 | 0.7211 | 0.7306 | 0.7344 | 1.3 | 1.8 |
| Solar zenith angle = 30°, $\tau_R = 0.2, \tau_{PS} = 0.2$ | | | | | | |
| Normalization constant = | | 1.0003 | 1.0652 | 1.0904 | 6.5 | 9.0 |
| 0 | 0.0 | 1.0000 | 1.0000 | 1.0000 | 0.0 | 0.0 |
| 4 | 2.0 | 0.9785 | 0.9797 | 0.9802 | 0.1 | 0.2 |
| 8 | 4.0 | 0.9183 | 0.9227 | 0.9245 | 0.5 | 0.7 |
| 12 | 6.0 | 0.8303 | 0.8393 | 0.8430 | 1.1 | 1.5 |
| 16 | 8.0 | 0.7284 | 0.7426 | 0.7485 | 1.9 | 2.8 |
| Solar zenith angle = 60°, $\tau_R = 0.1, \tau_{PS} = 0.2$ | | | | | | |
| Normalization constant = | | 1.4679 | 1.5860 | 1.6026 | 8.0 | 9.2 |
| 0 | 0.0 | 1.0000 | 1.0000 | 1.0000 | 0.0 | 0.0 |
| 4 | 3.5 | 0.9360 | 0.9399 | 0.9405 | 0.4 | 0.5 |
| 8 | 6.9 | 0.7774 | 0.7908 | 0.7930 | 1.7 | 2.0 |
| 12 | 10.4 | 0.5932 | 0.6167 | 0.6207 | 4.0 | 4.6 |
| 16 | 13.9 | 0.4347 | 0.4658 | 0.4714 | 7.2 | 8.4 |
| Solar zenith angle = 60°, $\tau_R = 0.2, \tau_{PS} = 0.2$ | | | | | | |
| Normalization constant = | | 1.2357 | 1.3683 | 1.3905 | 10.7 | 12.5 |
| 0 | 0.0 | 1.0000 | 1.0000 | 1.0000 | 0.0 | 0.0 |
| 4 | 3.5 | 0.9377 | 0.9429 | 0.9438 | 0.6 | 0.7 |
| 8 | 6.9 | 0.7833 | 0.8012 | 0.8043 | 2.3 | 2.7 |
| 12 | 10.4 | 0.6039 | 0.6343 | 0.6414 | 5.0 | 6.2 |
| 16 | 13.9 | 0.4494 | 0.4920 | 0.5001 | 9.5 | 11.3 |

creased (e.g., 40, instead of the 16 in this case). However, no definite conclusions can be drawn without making extensive test runs of the codes, which is beyond the scope of this paper.

### Effects of Multiple Scattering

We use our code to compute the SS and total radiance for different optical depths, solar zenith angles, aerosol characteristics, and wavelengths of incident solar radiation. An example of a typical input data is as follows. The aerosol size distribution[1,12] is a modified gamma distribution (MGD) given by

$$n(r) = ar^2 \exp(-br), \quad 0.03 \leq r \leq 2.0\,\mu\text{m}, \quad b = 10.0\,\mu\text{m}^{-1}, \quad (2)$$

the particle refractive index $m = 1.55$; volume scattering (and extinction) coefficients, $\beta_{sc}$ (and $\beta_{ext}$), are given by $\beta_{ext} = \beta_{sc} = 1.116 \times 10^8$ cm$^{-1}$ per particle; and $\lambda = 0.55$ $\mu$m. The results are given in Table IV for solar zenith angles 30° and 60°, and optical depth components for Rayleigh ($\tau_R$) and Mie ($\tau_{PS}$) particles, being (0.1, 0.2) and (0.2, 0.2), respectively.

The results presented in Table IV have been arranged to illustrate two points: the MS contributions to the absolute magnitude of the almucantar radiance, and the MS contributions to the shape of the almucantar radiance curve. Thus the first row in each of the six sections of the Table gives the actual radiance for a zero degree azimuth angle (and hence, zero degree scattering angle). Note that the incident flux was taken as $F = \pi$ for these computations. All other rows in each section give the normalized radiances, $I^N_{SS}$ and $I^N_{RT}$, where

$$I^N \equiv I(\psi)/I(0). \quad (3)$$

Finally, in the two right-hand columns, we give the percentage differences $(I^N_{RT} - I^N_{SS})/I^N_{SS}$, which serves to indicate the deviation in the shape of the almucantar radiance curve due to MS.

### Concluding Remarks

From the results given in Table IV, we see that MS contributes between 4.5% and 12.5% to the magnitude of the aureole radiance within the range of parameters we have considered. Of more concern to the retrieval of the size distribution from almucantar radiances, however, is the MS contribution to the shape of the almucantar radiance curve. Table IV suggests that provided we restrict ourselves to scattering angles of less than about 10°, the error in the shape of the curve is unlikely to exceed 3%. The azimuth range implied by this range of scattering angles is quite strongly dependent on solar zenith angle $\theta_{sun}$, as given by the formula connecting scattering angle $\psi$ and azimuth angle $\phi$, viz.,

$$\cos\psi = \cos^2\theta_{sun} + \sin^2\theta_{sun} \cos\phi \quad (4)$$

As mentioned in the Introduction, the effects of MS on the retrieval of the aerosol SD will be examined in a subsequent publication.

This work was supported by NASA contract NAS1-15198.

### References

1. A. E. S. Green, A. Deepak, and B. J. Lipofsky, Appl. Opt. **10**, 1263 (1971).
2. R. D. McPeters and A. E. S. Green, Appl. Opt. **15**, 2457 (1972).
3. J. T. Twitty, J. J. Parent, J. A. Weinman, and E. L. Eloranta, Appl. Opt. **15**, 980 (1976).
4. A. Deepak, "Inversion of Solar Aureole Measurements for Determining Aerosol Characteristics," in *Inversion Methods in Atmospheric Remote Sounding,* A. Deepak, Ed. (Academic, New York, 1977), pp. 297–323.
5. J. V. Dave, "Development of Programs for Computing Characteristics of U.V. Radiation," NASA contract 5-21680 (1972).
6. O. P. Bahethi and R. S. Fraser, "Effect of Molecular Anisotropy on the Intensity and Degree of Polarization of Light Scattered from Model Atmospheres," Goddard Space Flight Center, Greenbelt, Md., X-910-75-52.
7. J. G. Kuriyan, "Particulate Sizes from Polarization Measurements," The UCLA International Conference on Radiation and Remote Probing of the Atmosphere, August 1973, J. G. Kuriyan, Ed.
8. J. G. Kuriyan, D. H. Phillips, and R. C. Willson, J. R. Meteorol. Soc. **100**, 665 (1974).
9. J. V. Dave and P. H. Furukawa, "Scattered Radiation in the Ozone Absorption Bands at Selected Levels of a Terrestrial Rayleigh Atmosphere," Meteorological Monographs, Vol. 7, No. 29 (American Meteorological Society, Boston, 1966).
10. K. L. Coulson, J. V. Dave, and Z. Sekera, Tables Related to Radiation Emerging from a Planetary Atmosphere" (U. California, Berkeley, 1960).
11. B. M. Herman and S. R. Browning, J. Atmos. Sci. **22**, 559 (1965).
12. A. Deepak and G. P. Box, "Analytical Modeling of Atmospheric Aerosol Size Distributions," IFAORS-106-78 (in preparation).