

Atmospheric turbidity and the circumsolar radiation

M. A. Box, S.-Y. Lo, B. H. J. McKellar, and M. H. Reich

We investigate the possibility of determining the turbidity from the intensity of the circumsolar radiation and find that such a connection can be made only when the size distribution of the aerosol particles is known. However, measurements of both the turbidity and the aureole intensity can give useful information about the size distribution.

I. Introduction

In two recent papers Angstrom^{1,2} has presented the results of an examination of pyrheliometric and pyranometric records at two locations (Montezuma and Table Mountain) over the 1940–1950 period. On the basis of those observations Angstrom proposed that a relationship exists between the intensity of circumsolar radiation and the turbidity, permitting one to be determined from the other.

In this paper we will examine the theoretical basis for such a relationship. Both the intensity of circumsolar radiation and the turbidity depend upon the size distribution and the density of scatterers. We investigate the effect of changes in the size distribution upon the relationship between the aureole intensity and turbidity. The size distributions used are Deirmendjian's Haze H and a bimodal modification of it.

II. Theoretical Considerations

A. Turbidity Coefficient

The measure of turbidity that is used by Angstrom is the turbidity coefficient β_A . β_A is usually obtained through measurement of the optical thickness and then applying the relation

$$\tau_p(\lambda) = \beta_A(\lambda/\lambda_0)^{-1.3}, \quad (1)$$

where $\tau_p(\lambda)$ is the optical thickness at wavelength λ , and the reference wavelength λ_0 is 1μ .

In this paper we use a more general turbidity coefficient than Angstrom's β_A , where the particulate optical thickness is assumed to have an average wavelength exponent of 1.3. In fact the exponent α describing the wavelength dependence of the optical thickness by

$$\tau_p(\lambda) = \beta(\lambda/\lambda_0)^{-\alpha} \quad (2)$$

is a function of the particulate size distribution and, if measured, can provide information about the distribution. Since our Mie scattering theory calculations provide us with a theoretical value of α , we use β defined by Eq. (2) rather than β_A of Eq. (1). If we were to attempt the latter we would obtain different results for each different wavelength to the extent that α differs from 1.3.

Indeed we would emphasize that, if multispectral extinction measurements are available, the turbidity parameter of Eq. (2) should be extracted from the data together with the empirical value of α .³ This provides more information and can be used to determine properties of the particle distribution. Quoting extinction measurements in terms of the Angstrom turbidity coefficient is defensible only when multispectral measurements are not made, as was indeed the case for the measurements used by Angstrom in his analysis.

B. Aureole Intensity

The aureole intensity in a circumsolar band extending from θ_{\min} to θ_{\max} from the solar center can be found by integrating the scattered intensity over this scattering angle range and over the wavelength range of the solar radiation reaching the earth. The intensity of scattered radiation can be obtained from the equations of radiative transfer and, if multiple scattering and ground reflection effects are ignored, can be written as in the notation of Ref. 4:

$$B(\lambda) = H(\lambda) \exp(-\tau_o \sec \phi) \sec \phi [\tau_{ao} F_a R(\psi) + \tau_{po} F_p P(\tau)],$$

where B is the scattered radiance, $H(\lambda)$ is the solar irradiance, τ_o , τ_{ao} , and τ_{po} are the total, molecular, and particulate optical thicknesses. $R(\psi)$ and $P(\psi)$ are the respective molecular and particulate phase functions, and F_a and F_p are known as the molecular and particulate effectiveness factors.

The wavelength range of solar radiation that reaches the earth extends from the uv cut-off at $0.3 \mu\text{m}$ out to a series of ir absorption bands beyond $0.8 \mu\text{m}$. It is not possible to determine the exact effect of these bands on the solar aureole unless we have information about the

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levels of water vapor and carbon dioxide present in the atmosphere during the periods when the observations were made. Therefore we have not included the effect of the ir absorption bands in our calculation and instead have attempted to compensate for the neglect of these bands by suitably restricting the upper wavelength limit. It was found that an upper wavelength limit of $1.2 \mu\text{m}$ gave results that were compatible with the data presented in Angstrom's papers. The intensity in the circumsolar band extending from θ_{\min} to θ_{\max} was obtained by simply integrating the scattered intensity over the scattering angle range $\theta_{\min} \rightarrow \theta_{\max}$ and assuming that the zenith angle ϕ of each scattering element in the angular band was equal to the solar zenith angle ψ . This assumption results in an important simplification of the sky radiation equation, as it makes the scattered intensity independent of the altitude distribution of scatterers. It is possible to gauge the accuracy of this approximation by examining the behavior of the altitude distribution-dependent factor (often denoted as the effectiveness factor) as we vary the zenith angles of the sun and the scattering element. The particulate effectiveness factor F_p can be written as

$$F_p = \frac{1}{H_p(0)} \int_0^\infty \exp[-\tau(y)(\sec\psi - \sec\phi)] \left[-\frac{dH_p(y)}{dy} \right] dy,$$

where $H_p(y)$ is the integrated height of particulates above height y , and $\tau(y)$ is the total optical thickness above height y .

The data that were utilized by Angstrom were measured so that the value of $\sec\psi$ was 2, and $\theta_{\min} = 3.5^\circ$ and $\theta_{\max} = 13.5^\circ$. Therefore the value of the term $\sec\psi - \sec\phi$ was 1.32 and -0.52 at the extreme points A and B in Fig. 1. Green *et al.*⁴ have calculated the value of F_p for realistic models of the particulate altitude distribution and $\sec\psi = 2$. Their results indicate that F_p does not differ greatly from one at these points having a value of about 1.2 at point A and about 0.9 at point B. Thus our approximation appears reasonable.

C. Particulate Size Distribution

The particulate size distribution model that we use in this paper is one of a set of models that have been proposed by Deirmendjian of the form

$$n(r) = ar^\alpha \exp(-br^\gamma),$$

where $n(r)$ is the volume concentration of the particulates at radius r , and a , α , b , and γ are free parameters.

For aerosols Deirmendjian considered three particular cases, Haze H ($\gamma = 1, \alpha = 2$), Haze L ($\gamma = 1/2, \alpha = 2$), and Haze M ($\gamma = 1/2, \alpha = 1$). These models have an advantage over Junge distribution models because they, unlike the Junge models, provide zero particle densities when the particle radius is zero. Thus a further parameter, a lower limit cutoff in the size distribution, is not required to exclude the origin when integrating over the size distribution range.

Kuriyan *et al.*⁵ have shown that the three Deirmendjian Hazes, H , L , and M , are redundant as the intensity of radiation scattered near the forward direction from Hazes L and M can be duplicated by the scattering from

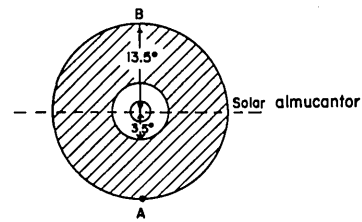


Fig. 1. The shaded annulus shows the region over which the circumsolar radiation was measured in the experiments reported by Angstrom. The points A and B represent the farthest deviation from the almucantor condition $\psi = \phi$.

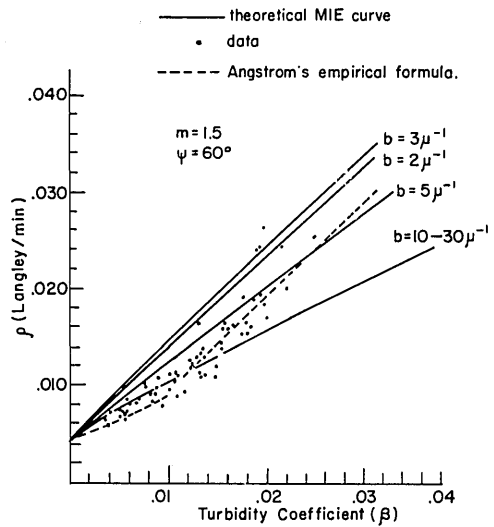


Fig. 2. The theoretical intensity plotted against the turbidity coefficient for different values of the size distribution parameter b . Also shown are Angstrom's empirical formulas and the data from which the formulas were obtained.

Haze H if the model parameters are chosen correctly. Therefore we have used only the Haze H model or combinations of Haze H distributions as our particulate size distribution model in this paper.

III. Results and Discussion

In Fig. 2 the results of our calculations of the aureole intensity using Mie theory are shown plotted against β for different values of b in the range $b = 2 \mu\text{m}^{-1}$ to $b = 30 \mu\text{m}^{-1}$ for a refractive index of 1.50. We note that for $10 \mu\text{m}^{-1} \leq b \leq 30 \mu\text{m}^{-1}$, i.e., small particles, the curves all virtually coincide. However, the curves for smaller values of b ($b < 10 \mu\text{m}^{-1}$), or larger particles, differ, being steeper by varying amounts than the small particle distribution curves. Also plotted in Fig. 2 are the experimental data obtained from Angstrom's paper. For $\beta < 0.015$ the curves with $10 \mu\text{m}^{-1} \leq b \leq 30 \mu\text{m}^{-1}$ pass through the data while for larger β the data tend to be above these curves and nearer to the curves with $b < 10 \mu\text{m}^{-1}$. This indicates that as the turbidity increases, either the modal size of the distribution increases or a

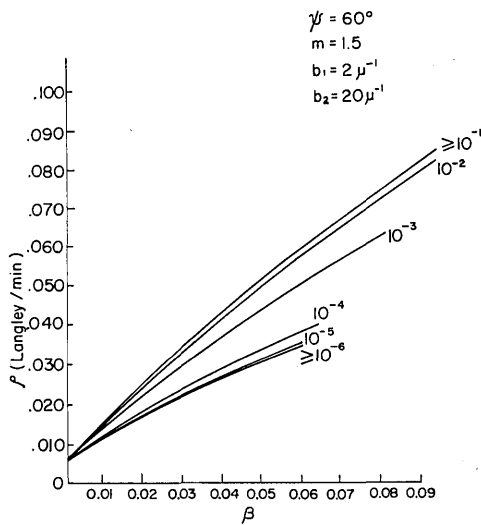


Fig. 3. The theoretical intensity produced by the scattering from a two component particulate size distribution plotted against the scattering coefficient β , for various values of the ratio h_1/h_2 , h_1 and h_2 are maximum volume concentrations of large and small particle distribution, respectively.

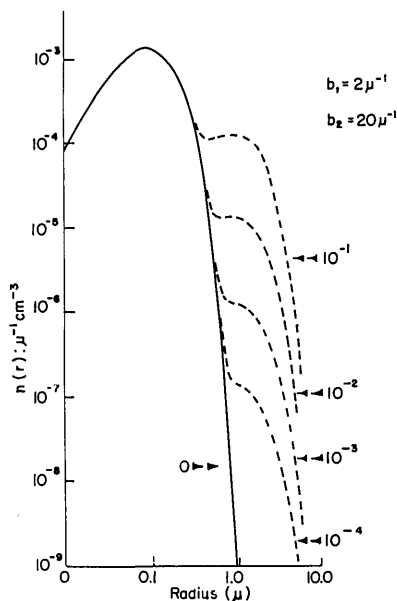


Fig. 4. The particulate size distributions that correspond to the aureole intensities in Fig. 3. The numbers shown against the various curves are the values of the ratio h_1/h_2 .

Table I. Range of Turbidities

$\rho (\times 10^3)$: langley/min	Min β	Max β
5	0.0006	0.0012
10	0.0058	0.0095
15	0.0106	0.0186
20	0.0160	0.0280
25	0.0214	0.0380

significant large particle component becomes present in the size distribution (significant in that there are enough large particles present so that the aureole scattering thus produced swamps the scattering due to the main peak in the size distribution). To decide between these possibilities we note that previous studies^{6,7} of the aerosol size distribution have indicated that the main peak in the size distribution usually occurs at radii less than $0.2 \mu\text{m}$ ($b \geq 10 \mu\text{m}^{-1}$). Moreover evidence of a small but significant large particle component in the size distribution has been found at various locations.^{8,9} We therefore reject the first possibility in favor of the second.

Stating this conclusion another way, we can say that the data presented in Angstrom's papers indicate that the increase in turbidity past $\beta = 0.015$ is probably caused by an increase in the relative number of large particles present, rather than just an over-all increase in the total number of particles present.

We can examine the effect of this large particle component more closely by seeing how the curves behave as we vary the large particle loading relative to the particle loading of the main peak in the size distribution. Figure 3 shows the aureole intensity plotted against β for various amounts of a $b = 2\text{-}\mu\text{m}^{-1}$ distribution added to a larger amount of a $b = 20\text{-}\mu\text{m}^{-1}$ distribution. The corresponding size distributions are shown in Fig. 4. We can see from these two figures that when the ratio of the large particle component maximum to the small particle component maximum is less than $\sim 10^{-4}$, the relationship between the aureole intensity and turbidity is approximately the same as that of the single component $b = 20\text{-}\mu\text{m}^{-1}$ case. When the ratio is 10^{-2} or greater the curve behaves like the $b = 2\text{-}\mu\text{m}^{-1}$ case while in the transition region the curve interpolates between the $b = 20\text{-}\mu\text{m}^{-1}$ and $b = 2\text{-}\mu\text{m}^{-1}$ cases. The ratio of the large particle maximum to the small particle maximum is $\sim 10^{-3}$ when the aureole intensity curve is half way between the two limiting cases.

From our calculations we conclude that measurement of the aureole intensity in a 10° wide band concentric with the sun will not provide a unique measure of the turbidity. In the absence of prior information as to the type of particulate size distribution present in the atmosphere, all we can say, given the aureole intensity, is that the turbidity is anywhere between the small ($b > 10\text{-}\mu\text{m}^{-1}$) and large particle ($b = 2\text{-}\mu\text{m}^{-1}$) limits. Its precise value depends on the nature of the size distribution. In Table I we present the range of possible turbidities for various aureole intensities. We can see from this table that if the aureole intensity is measured to be, say 0.02 langley/min, in principle there is an uncertainty of $\sim 30\%$ in the turbidity. Specifically β could be anywhere in the $\sim 0.016 \rightarrow \sim 0.028$ range depending on whether there was a significant large particle component present. Similarly for other aureole intensities the uncertainty is of the same order. Hence we can only ascribe Angstrom's more or less unique empirical relation to the fact that the size distribution at the location of measurements varied over a limited range and suggest that the same relationship may not hold at other locations.

We conclude by emphasizing that, without information about the size distribution, we cannot determine the turbidity from measurements of the aureole. However, measurements of both turbidity and aureole intensity can provide information on the particle size distribution. This is especially true if the turbidity measurements are the result of multispectral extinction type, so that an empirical value of α is also available.

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References

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|-------------|--|-------------|--|
| 7 | Florida Section OSA Meeting, Winter Park <i>N. Mohon, OSA Florida Section, 3307 Balsam Dr., Winter Park, Fla. 32792</i> | 13-17 | 32nd Annual Symposium on Molecular Spectroscopy, Columbus <i>K. N. Rao, Dept. of Phys., Ohio State Univ., 174 W. 18th Ave., Columbus, Ohio 43210</i> |
| 8-13 | Electrochemical Society, mtg., Philadelphia <i>P. L. Castro, Hewlett Packard Labs., 3500 Deer Creek Rd., Palo Alto, Calif. 94304</i> | 20-23 | Canadian Association of Physicists Congress, Saskatoon <i>M. Jento, CAP, 151 Slater St., Ottawa, K1P 5H3.</i> |
| 10-12 | Thermophysical Properties, symp., Gaithersburg, Md. <i>A. Cezairliyan, Room 124 HAZ Bldg., NBS, Washington, D.C. 20234</i> | 20-24 | Laser/Electrooptics, conf., Munich <i>Munich Fair Authority, D-8000, Munich 12, POB 12 10 09, Messege-laende, West Germany</i> |
| 17-19 | 23rd ISA Analysis Instrumentation Symposium, Charleston <i>E. G. Trawick, Union Carbide Corp., Box 8361, S. Charleston, W. Va. 25303</i> | 20-24 | Advances in Color Technology, course, Troy <i>Office of Continuing Studies, Rensselaer Polytechnic Instit., Troy, N.Y. 12181</i> |
| 23-25 | Plasma Science, internat. conf., Troy <i>R. P. Hickok, Rensselaer Polytechnic Instit., Troy, N.Y. 12181</i> | 20-24 | APCA Annual Conference and Exhibition, Toronto <i>Public Relations Dept., Air Pollution Control Assoc., P.O. Box 2861, Pittsburgh, Pa. 15230</i> |
| 30-3 June | Canadian Conference of Applied Mechanics, Vancouver <i>C. R. Hazell, CANCAM 77, Faculty of Applied Science, Univ. of British Columbia, Vancouver, B.C., Canada</i> | 20-24 | International Conference on Fourier Transform Infrared Spectroscopy. Columbia, S.C. <i>J. W. Quinn, OSA, 2000 L St. N.W., Washington, D.C. 20036</i> |
| 30-3 June | Spectacle Lens Technology, course, Cranfield <i>Registrar, Cranfield Inst. of Technol., Cranfield, Bedford MK43 OAL., U.K.</i> | 21-23 | Machine Processing of Remotely Sensed Data, symposium, W. Lafayette <i>J. C. Lindenlaub, Lab. for Applications of Remote Sensing, Purdue Univ., 1220 Potter Dr., W. Lafayette, Ind. 47906</i> |
| June | | 21-24 | Matrix Isolation Spectroscopy, mtg., Berlin <i>H. Gerischer, Ritz-Haber-Instit., MPG, 1 Berlin 33, Faradayweg 4-6, West Germany</i> |
| ? | American Astronomical Society, mtg., Atlanta <i>L. W. Fredrick, Box 3818, Univ. Sta., Charlottesville, Va. 22903</i> | 27-30 | International Research Group on Color Vision Deficiencies, symp., Parma <i>M. Maione, Clinica Oculistica dell'Universita, I-43100 Parma, Italy</i> |
| 1-3 | Laser Engineering Applications, conf., Washington <i>J. W. Quinn, OSA, 2000 L St. N.W., Washington, D.C. 20036</i> | July | |
| 6-8 | Pattern Recognition and Image Processing, conf. <i>H. Freeman, Electr. and Systems Eng. Dept., Rensselaer Polytechnic Instit., Troy, N.Y. 11281</i> | 4-9 | International Congress on Acoustics, Madrid <i>Sociedad Espanola de Acustica, Serrano, 144, Madrid, 6, Spain</i> |
| 6-9 | Laser Atmosphere Studies, ASTM Comm. 8th Conf., Philadelphia <i>J. Cooney, Drexel Univ., Philadelphia, Pa. 19104</i> | 6-8 | Hot Electrons in Semiconductors, internat. conf., Denton <i>D. G. Seiler, North Texas State Univ., Physics Dept., Denton, Texas 76203</i> |
| 6-9 | International Magnetism Conference, Los Angeles <i>IEEE, 345 E. 47th St., New York, N.Y. 10017</i> | 10-15 | International Color Association Congress, Troy <i>F. W. Billmeyer, Jr., Dept. of Chem., Rensselaer Polytechnic Instit., Troy, N.Y. 12181</i> |
| 6-10 | Principles of Color Technology, course, Troy <i>Office of Continuing Studies, Rensselaer Polytechnic Instit., Troy, N.Y. 12181</i> | 17-21 | Spectroscopy and Analytical Chemistry, conf., Birmingham <i>P. E. Hutchinson, Analy. Div. TCS 9/10 Saville Row, London W1X 1AF, U.K.</i> |
| 7-10 | Acoustical Society of America Spring Meeting, State College <i>J. C. Johnson, Applied Res. Lab., Penna. State Univ., State College, Pa. 16802</i> | 18-20 | International Conference on Integrated Optics and Optical Fiber Communication (IOOC), Tokyo <i>T. Okoshi, Dept. of Electrical Eng., Univ. of Tokyo, 7-3-1 Hongo, Bunkyo-Ku, Tokyo 113, Japan</i> |
| 8-10 | Conference on Coherence and Quantum Optics, Rochester <i>E. Wolf, Physics Dept., Univ. of Rochester, River Campus Station, Rochester, N.Y. 14627</i> | | |
| 13-14 | Color Technology for Management, course, Troy <i>Office of Continuing Studies, Rensselaer Polytechnic Instit., Troy, N.Y. 12181</i> | | |

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