An efficient method to increase vertical resolution of actinic flux calculations in clouds

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[1] The errors associated with using different vertical resolutions for broadband actinic flux calculations have been investigated for the case of a boundary layer cloud. Results are presented for 10, 20, and 40 layers in the lowest kilometer of the atmosphere, with comparisons made against a benchmark of 100 layers. A more accurate picture of actinic flux profiles is obtained by increasing the number of vertical layers. We also derive an expression for the average value of actinic flux within a computational layer. This result, when used to quadratically interpolate the coarser layerings, improved the accuracy in the 10-layer case by a factor of five, when compared with a simple linear interpolation between the original data points. *INDEX TERMS:* 0320 Atmospheric Composition and Structure: Cloud physics and chemistry; 0360 Atmospheric Composition and Structure: Transmission and scattering of radiation; 0305 Atmospheric Composition and Structure: Aerosols and particles (0345, 4801); 3359 Meteorology and Atmospheric Dynamics: Radiative processes; *KEYWORDS:* Actinic flux, clouds, photochemistry, radiation, heating rates

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1. Introduction

[2] Solar (and terrestrial) radiation drives many of the physical and chemical processes which take place in the Earth's atmosphere, including photochemistry [*Los et al.*, 1997] and cloud heating/cooling rates, and (indirectly) cloud microphysical evolution [*Hu and Stamnes*, 1993]. Near the top and bottom of cloud layers, some radiation quantities, such as actinic flux, vary quite rapidly in the vertical [*Madronich*, 1987], so that a high vertical resolution is required of the radiative transfer calculations if such gradients are to be accurately captured. This is especially important when modeling feedbacks between radiant energy and cloud microphysics.

[3] In this paper, we examine the errors involved in calculating the broadband actinic flux profiles using different vertical resolutions in the case of a boundary layer cloud. Results for 10, 20, and 40 layers in the lowest kilometer are compared with those for 100 layers (taken as our benchmark). In order to make this comparison, we will linearly interpolate the results from the fewer layer cases, and it is the errors involved in these interpolations that will be our initial focus.

[4] In addition, we derive an expression for the average value of the actinic flux within a computational layer. This then provides an additional piece of information on how actinic flux varies within a given layer. Using this information we are able to quadratically interpolate between layer boundaries, significantly reducing the errors, especially in the case of the coarsest vertical resolution.

2. Actinic Flux Calculations

[5] The actinic flux is defined as the radiant energy density incident onto a spherical unit area [*Ruggaber et al.*, 1993]:

$$AF_\lambda(z) = \int_{4\pi} d\Omega I_\lambda(z,\Omega). \tag{1}$$

The actinic flux considers all the radiation at a point, not just the vertical component. It is an important photochemical quantity as photodissociation rate coefficients are calculated by integrating the product of the spectral actinic flux, spectral absorption cross section, and the photodissociation quantum yield over all relevant wavelengths [*Madronich*, 1987]. Heating and cooling rates may also be calculated via the actinic flux (Note that our investigations are for the full solar spectrum).

[6] In contrast, the net flux is defined as the integration of the normal component of the intensity, I_λ, over the solid angle, $d\Omega$, at a given height z:

$$F_\lambda(z) = \int_{4\pi} d\Omega I_\lambda(z,\Omega)\mu, \tag{2}$$

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where $\mu = \cos\theta$ and θ is measured between the element of solid angle and the normal to the area through which the radiation is passing. It is important for the flow of radiant energy.

[7] Standard computer codes return intensity as a function of direction at the boundaries between computational layers. From this information quantities such as flux and actinic flux may be readily calculated. However, it is also possible to use these data to compute the average value of actinic flux within a computational layer. We start by writing the equation of radiative transfer in the form:

$$\mu \frac{dI_\lambda}{d\tau} = I_\lambda - \frac{\tilde{\omega}}{4\pi} \int p(\Omega' \to \Omega)\, I_\lambda(\Omega') d\Omega', \tag{3}$$

where $\tilde{\omega}$ is the single scattering albedo, p is the phase function, τ is the optical thickness, and I_λ is the (spectral) intensity (Here I_λ represents the total intensity, diffuse plus direct beam, with appropriate boundary conditions). Integrating both sides of equation (3) with respect to Ω yields:

$$\frac{d}{d\tau} \int \mu\, I_\lambda(\Omega)\, d\Omega = \int I_\lambda(\Omega)\, d\Omega - \frac{\tilde{\omega}}{4\pi} \int \int p(\Omega' \to \Omega) \cdot I_\lambda(\Omega')\, d\Omega'\, d\Omega. \tag{4}$$

The left-hand side of equation (4) can be simplified by making use of equation (2) (i.e., the definition of flux). The right-hand side of equation (4) is simplified by using equation (1) (the definition of actinic flux) and the normalization of the phase function. Equation (4) can therefore be simplified to

$$\frac{d}{d\tau} F_\lambda = AF_\lambda - \tilde{\omega} \int I_\lambda(\Omega')\, d\Omega' = AF_\lambda(1 - \tilde{\omega}). \tag{5}$$

Therefore the average actinic flux within a layer of optical thickness $\Delta\tau$ is

$$AF_\lambda = \frac{\Delta F_\lambda}{(1 - \tilde{\omega})\Delta\tau} = \frac{\Delta F_\lambda}{\Delta\tau_{abs}}. \tag{6}$$

We shall refer to this result as the flux difference method.

3. Model Parameters

[8] The model employed to perform the spectral integration is based on the correlated *k* distribution method [*Fu and Liou*, 1992], which has the advantage of computational efficiency. Our model is run with six solar bands. Spectral integration of equation (1) is then performed by simply adding the contributions from each subband. The atmospheric model allows for gaseous absorption by water vapor, ozone, methane, nitrous oxide, and carbon dioxide. The atmospheric profile is that of Midlatitude Summer, except that the relative humidity profile we have used has a constant humidity up to 4 km and zero humidity above this altitude. The relative humidity in the lowest 4 km was set at 99% during this study. This profile was chosen to make our results easily reproducible.

[9] The δ-four stream approximation has been used, which relies on the general solution for the discrete ordinates method of radiative transfer, which becomes partially analytic for four streams. *Zeng et al.* [1996] investigated actinic flux values using a two-stream approximation within clouds. From previous investigations [*Kay et al.*, 2001] we found that the two-stream approximation we used had considerable errors at the cloud base and top, whereas the δ-four stream approximation had negligible errors. Therefore the δ-four stream approximation was chosen, as it is a good compromise in terms of accuracy and efficiency for both actinic flux and net flux calculations [*Kay et al.*, 2001]. The aerosol type we have used is maritime, and it has been defined in *Shettle and Fenn* [1979] and reproduced by *d'Almeida et al.* [1991] [see also *Kay and Box*, 2000]. The maritime aerosol has been coupled to the appropriate surface albedo model: an ocean surface [*Stephens*, 1994; *Kay and Box*, 2000]. Modeling assumes a plane parallel atmosphere and a solar zenith cosine of 0.5.

[10] Clouds are generally classified according to their position and appearance in the atmosphere. We have looked at a Stratus I (ocean) cloud [*Liou*, 1992], with a fixed effective radius of 10.0 μm and a fixed liquid water content of 0.24 g m^{-3}. We have chosen the cloud base to be positioned at 0.4 km and cloud top at 0.8 km, parameters typical for marine boundary layer clouds.

[11] The comparisons between varying the number of vertical layers in the lowest kilometer are for 10, 20, and 40 equally spaced layers. A benchmark is needed as the case against which all other vertical resolution results will be compared. The case for 100 layers is used as the benchmark.

4. Results

4.1. Layer Average Values

[12] The derivation of equation (6) shows that the actinic flux value obtained by the flux difference method will be the average value through the layer in question. To test this conclusion, we have compared the average values obtained in this way against the numerical average obtained by integrating all contributions through the given layer from the 100 layer runs. The "true" average value is based on the 100 layer results. This therefore incorporates the 10 points from the 100 layers that correspond to the number of points in 0.1 km for the case of 10 vertical layers in the lowest kilometer, and 5 points that correspond to the number of points in 0.05 km for the case of 20 vertical layers in the lowest kilometer. The results for the averaging in the case of the 100 layers are considered quite accurate, as the greater the number of points, the more accurate the results should be. We have also compared these results with the value obtained by taking a simple layer mean, which is the average of the two layer boundary values.

[13] Table 1 displays the average values based on 100 layers (our best estimate of "truth"), the estimated average from equation (6), and the simple layer mean, for both the 10 and 20 layer calculations. As these results show, the estimated average value (flux difference method) and the average (100 layer) value are extremely close throughout the cloud layer, with only minimal differences near the cloud boundaries. By comparison, the layer mean values show a clear difference from the other two results. These results show that the flux differ-

Table 1. Actual Average Values of Actinic Flux and Estimated Values for 10 and 20 Layers

Altitude for Flux Difference Method, km	Average Flux[a]	Estimated Average Flux[b]	Layer Mean
	10 Layers		
0.35	315.51	315.51	315.48
0.45	473.40	473.54	462.45
0.55	728.10	728.09	728.90
0.65	992.09	991.97	1001.19
0.75	1459.35	1459.09	1520.79
0.85	1902.39	1902.39	1902.22
	20 Layers		
0.375	318.48	318.48	318.47
0.425	405.66	405.93	399.64
0.475	541.14	541.16	540.68
0.525	665.49	665.49	665.59
0.575	790.71	790.70	791.00
0.625	920.14	920.10	921.07
0.675	1064.03	1063.86	1067.96
0.725	1267.66	1267.03	1282.77
0.775	1651.04	1651.18	1655.49
0.825	1898.04	1898.04	1898.00

[a] Average values are based on 100 layers.
[b] Estimated average values are based on the flux difference method.

ence method provides an accurate value for the average actinic flux within a layer.

4.2. Actinic Flux Profiles

[14] The first comparison to make is between the original calculation of actinic flux for the three vertical resolutions (10, 20, and 40 layers), versus our benchmark (100 layers): This is displayed in Figure 1. Actinic flux values were linearly interpolated for the 10, 20, and 40 layer runs, so that there were the same number of points (100) to graph in each case. The most obvious difference is for the case of 10 vertical layers, between ~0.7 and 0.8 km. Here a vertical resolution of 0.1 km cannot include the curvature that the 100 layer result shows. To quantitatively determine the difference between each case and the benchmark, a difference term is defined:

$$Difference = AF_{\text{no. of layers}} - AF_{100\,\text{layers}}. \tag{7}$$

[15] The inset to Figure 1 displays the difference term for the original actinic flux calculations. The difference from the benchmark is most noticeable just inside the boundaries of the cloud layer, near 0.4 and 0.8 km. A resolution of ten vertical layers in the lowest kilometer clearly shows the greatest differences (errors), being five times less accurate than a resolution of twenty vertical layers in the lowest kilometer. A resolution of 40 layers in the lowest kilometer is extremely accurate, with almost no visible difference with respect to a resolution of 100 layers.

[16] Figure 1 shows us that actinic flux can vary nonlinearly, at least within thick layers, so that linear interpolation is unlikely to yield results of sufficient accuracy. However, as we now have an additional piece of information, namely the layer average, we can use a higher order interpolation. While there are many potential interpolating functions, we will restrict our attention to the simplest, namely a quadratic. Thus we assume that, within

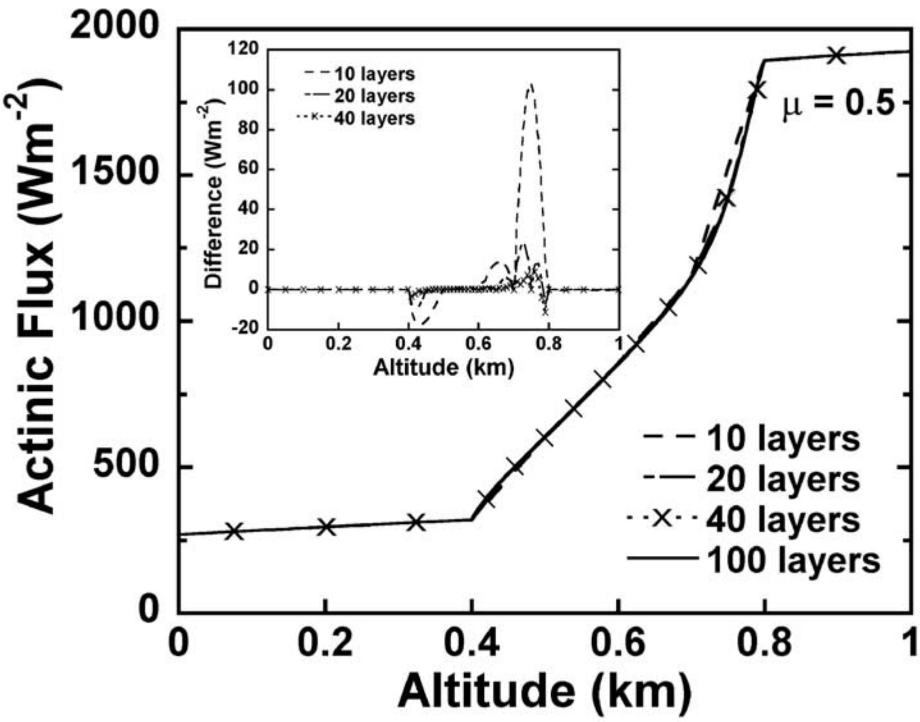

Figure 1. Vertical profile of actinic flux for 10, 20, 40, and 100 layers in the lowest kilometer using linear interpolation. The inset is the difference term.

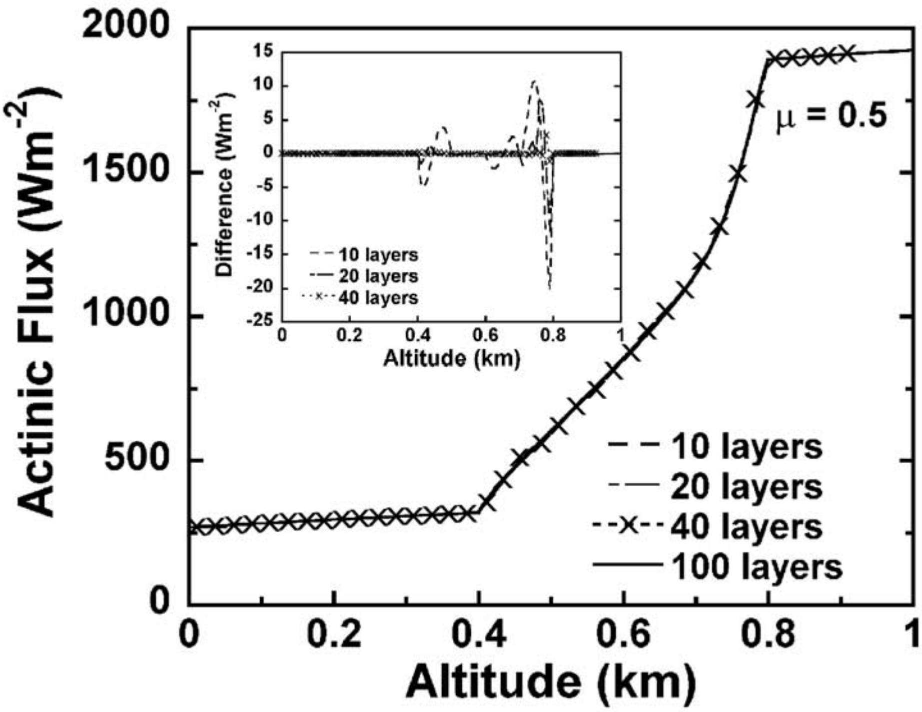

Figure 2. Vertical profile for the quadratically interpolated actinic flux for 10, 20, 40, and 100 layers in the lowest kilometer. The inset is the difference term.

a given layer, actinic flux varies with altitude according to:

$$AF = \alpha x^2 + \beta x + \gamma, \tag{8}$$

where x is a scaled coordinate, which varies from 0 at the bottom of a layer to 1 at the top. The actinic flux values at the top and bottom of the layer provide two pieces of information, and the third piece of information is provided by the layer average, which we know with high accuracy from equation (6), from which all 3 unknowns in equation (8) may be solved for:

$$\overline{AF} = \int_0^1 \left(\alpha x^2 + \beta x + \gamma\right) dx = \frac{1}{3}\alpha + \frac{1}{2}\beta + \gamma. \tag{9}$$

[17] Figure 2 displays quadratically interpolated actinic flux against altitude for the three different vertical resolutions, along with the benchmark results for 100 layers. The inset shows the difference term. When compared with the results for linear interpolation presented in Figure 1, we see a marked increase in accuracy. This is especially true in the case of 10 layers, the coarsest vertical resolution, where the maximum difference (error) has dropped from 100 W m^{-2} to 20 W m^{-2}. The maximum difference in the 20 layer case has decreased by a factor of 2.

5. Discussion

[18] There have been numerous studies involved in determination of the effects of vertical resolution within clouds (*Lane et al.* [2000], *Bushell and Martin* [1999], *Räisänen* [1999], and *Junkermann* [1994], to name a few). These investigations have been either into the errors involved using a different number of layers or investigating the vertical resolution of different parameters such as the water vapor or temperature profile. Our work has not just investigated the changes in accuracy by varying the number of vertical layers, it has also looked at an alternate way of determining the actinic flux, which can be used to give far more accurate values within a layer than simple linear interpolation.

[19] The flux difference method clearly provides us with the average value of actinic flux throughout a computational layer, as is demonstrated by the results in Table 1. For a resolution of 10 layers in the lowest kilometer, an error of <0.1% resulted between the estimated average and the 100 layer average value. The errors between the 100 layer average value and the simple layer mean values near cloud top and base were ~4.5 and 2.5%, respectively.

Table 2. Maximum Errors for Different Liquid Water Contents for Quadratic and Linear Interpolation

	1/2 LWC	LWC	2 × LWC
		10 Layers	
Quadratic	11.50	20.03	45.79
Linear	23.00	103.34	237.03
		20 Layers	
Quadratic	3.35	11.50	15.50
Linear	10.2	23.28	94.86
		40 Layers	
Quadratic	0.57	2.80	8.70
Linear	5.75	11.13	21.50

[20] The results presented so far are for a single cloud model: How might our conclusions be translated to other cases? Because optical depth is far more important in radiation transport than physical distance, we repeated our investigations for clouds with double, and half, the liquid water content (LWC). This had the effect of doubling, and halving, the scattering coefficient and hence optical thickness for a given physical layer. The same layering options were again used.

[21] In Table 2 we present the maximum error in all 9 cases for both quadratic and linear interpolation. We see from these results that, to a good approximation, the effect of doubling (halving) the LWC is the same as the effect of halving (doubling) the number of layers, that is, of doubling (halving) the layer spacing. This is to be expected. Thus we see that it is the cloud optical thickness in a given layer that dictates the accuracy of both interpolation methods. In all cases, we note that quadratic interpolation is superior to linear by at least a factor of 2.0, and often more (especially so in the optically thicker cases).

[22] It is always possible to improve the accuracy of actinic flux profiles by increasing the number of vertical layers. However, this may not be possible in practice because of time (and other) constraints, especially within chemical transport models. In this paper we have shown that the extra information provided by the layer average allows us to quadratically interpolate actinic flux within a computational layer. This improved the accuracy in the coarsest layering (0.1 km) by a factor of 5, and by a factor of 2 in the case of 20 layers. These improvements are available at essentially no cost.

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