Actinic Flux and Net Flux Calculations in Radiative Transfer—A Comparative Study of Computational Efficiency

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ABSTRACT

The accuracy and speed of three well-known computational techniques (DISORT, the δ–four-stream approximation, and the two-stream approximation), and the matrix inversion method, which is less well known, have been investigated. Results are presented for both broadband actinic and net fluxes over a range of parameters including solar zenith cosine, relative humidity, and altitude for two different surface/aerosol systems: terrestrial and oceanic. The matrix inversion method can only calculate actinic fluxes; therefore, this is the main focus of this paper. Investigations into the comparative accuracy of the four techniques for the oceanic model with and without a cloud layer included are also presented. (DISORT is taken as the benchmark for this research.)

Based on results presented here, it is found that for actinic flux calculations, the δ–four-stream approximation is slightly more accurate than the matrix inversion method, and that both are far more accurate than the two-stream approximation. However, for net flux calculations, the δ–four-stream approximation fares better and is clearly the most accurate. The superiority of the δ–four-stream approximation is particularly noticeable for both net and actinic fluxes when a cloud layer is included. In this paper, information is provided to assist modelers in choosing a computational technique that best suits their needs. The relative computational efficiency of the various radiative transfer techniques is also discussed for the benefit of those modelers who seek a compromise between time and accuracy, rather than solely maximal accuracy in a particular technique.

1. Introduction

Theoretical models of the scattering of radiation in the earth's atmosphere often require the solution of a fundamental equation that handles the transfer of radiation in planetary atmospheres: the radiative transfer equation (Chandrasekhar 1960). Due to the complexity of this equation, it is generally solved numerically rather than analytically. Various numerical methods for solving the radiative transfer equation have been under investigation for many years (Chandrasekhar 1960; Stamnes and Swanson 1981; Lenoble 1985; Stamnes et al. 1988a; Liou 1992, and numerous others), each involving a different compromise between speed and accuracy. While calculation time has largely decreased due to significant improvements in computing technologies in recent decades, it is also possible to streamline the calculation techniques within a single computing technology using various approximation schemes. It is the use of these approximation schemes, however, that may lead to losses in acceptable accuracy. The ''best'' method is often a compromise between these factors—time and accuracy.

Due to the large amount of computation needed for applications such as photochemical and climate modeling, determination of the accuracy and speed of various techniques is important when looking for ways of minimizing errors in calculations caused by different treatments of various parameters, but also for a method that is not computationally too intensive. Investigations of this type have been performed by various groups

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(King and Harshvardhan 1986; Boucher et al. 1998) to compare the accuracy of different methods for a specific quantity and to try to find out where the possible sources of error could be arising within the methods. King and Harshvardhan (1986) compared the doubling method to selected radiative transfer approximations, and also investigated the accuracies of asymptotic theory for thick layers, for monochromatic calculations. Boucher et al. (1998) compared 15 models to examine broadband and 550-nm partial radiative forcing by sulfate aerosol for a range of parameters. They were interested in the discrepancies between the models arising from the different treatment of calculations within the model.

This paper is a study of the relative computational efficiency of four commonly used approximation schemes: the discrete ordinates method (DISORT; Stamnes and Swanson 1981; Lenoble 1985; Stamnes et al. 1988a,b), (which we have run with 32 streams), the δ–four-stream method (Liou 1992), the two-stream method (Zdunkowski et al. 1980), and the matrix inversion method, which is based on work originally published by Meier et al. (1982).

There are many variations to the two-stream approximation (Meador and Weaver 1980; Edwards 1996) and all conclusions drawn from our work are for the particular version we have employed (Zdunkowski et al. 1980). Improvements can be made to the efficiency of the matrix inversion method if the only parameter that varies is the solar zenith cosine as no additional calculations need to be made, that is, reinverting the matrix. The δ–four-stream approximation (Liou et al. 1988) is widely known and documented, and has been proven to be relatively accurate.

The 32-stream method was used as the benchmark for assessments of the relative accuracy of the other three methods investigated. This is because it has been shown that when a greater number of streams is used, the results are generally more accurate (Box et al. 1993). While we will present results for both (spectrally integrated) actinic flux and net flux in realistic atmospheres, our major focus will be on the actinic flux as it is the key quantity for computing radiative heating rates and photodissociation frequencies.

Section 2 outlines the computational details and includes definitions of actinic and net fluxes, as well as the model parameters. The exact form of the radiative transfer equation depends on the various parameters that describe the model atmosphere. Hence, in section 3 we present the results of calculations for two basic physical situations: a maritime aerosol over an ocean surface, and a rural aerosol (with added soot) over a vegetation surface. For each combination, the actinic flux and net flux have been investigated with respect to varying solar zenith cosine, relative humidity, and altitude. The effects of clouds for the case of an ocean surface and maritime aerosol have also been investigated for both net and actinic flux. Section 4 presents a summary and discussion of results. Our conclusions and recommendations based on these results are presented in section 5.

2. Computational details

a. The computational methods

As noted above, four methods for solving the radiative transfer equation are employed in this paper. The key features of these methods are discussed here, commencing with the benchmark method DISORT (in our case employing 32 streams). More extensive discussions of the methods may be found in the references.

1) THE DISCRETE ORDINATES METHOD

The discrete ordinates method is one of the most frequently used techniques to solve the radiative transfer equation (Stamnes and Swanson 1981). It was originally developed by Chandrasekhar (1960) to study the transfer of radiation in planetary atmospheres, and has become a useful tool in calculating radiation in atmospheres containing aerosols and clouds (Stamnes et al. 1988a,b). This technique solves the radiative transfer equation via the discretization of the basic equation, representing it as a set of $2N$ coupled differential equations. These are solved using eigenanalysis.

2) THE DELTA (δ)–FOUR-STREAM APPROXIMATION

The δ–four-stream approximation is based on the same general solution for the discrete ordinates method for radiative transfer. However, with only four streams, a quadratic is solved instead to obtain the eigenvalues. Liou et al. (1988) present a comparatively straightforward solution that computes the solar radiative fluxes in an accurate manner. The accuracy of this method is enhanced by the incorporation of a delta-function adjustment, to account for scattering in the forward diffraction peak (see below).

3) THE TWO-STREAM APPROXIMATION

The two-stream approximation is one of the simplest techniques used to solve the radiative transfer equation. This method divides the multiple scattering contribution into two components, and upward and downward fluxes are calculated by solving two coupled differential equations. There are many variations of this method (Meador and Weaver 1980; Edwards 1996, and numerous others), and we have chosen to use the practical improved flux method, developed by Zdunkowski et al. (1980).

4) THE MATRIX INVERSION METHOD

The matrix inversion method is based on work originally published by Meier et al. (1982). The only quantity this method is able to calculate is the actinic flux.

TABLE 1. Computational time for the four methods.

Computational technique	C_T no clouds (44 layers)	C_T clouds (59 layers)
Two-stream approximation	0.4	0.6
Matrix inversion method	1.5	3.1
δ–four-stream approximation	2.1	2.9
32-stream method	85.6	109.8

This method makes use of the so-called source function (S) model (Sze 1976). For this model, the source function can be defined as the integral of the intensity over all directions. The intensity field (and hence the net flux) is not directly calculated in this method. However, by assuming that scattering processes are isotropic, there is a symmetry between ''incoming'' radiation (actinic fluxes) and ''outgoing'' (scattered) radiation. This symmetry is exploited to construct a matrix equation to directly compute the actinic flux in each layer. When applied to aerosol scattering, a delta-function correction is employed.

The matrix inversion method of Meier et al. (1982) has been enhanced by Becker et al. (2000) by implementing an improved treatment of the diffuse radiation field. Plumb and Ryan (1998) based their treatment of the effects of multiple scattering on Meier et al. (1982), and accounted for details omitted for the expression of the source function. The version of the matrix inversion code we have used is that of I. Plumb from the Commonwealth Scientific and Industrial Research Organisation (CSIRO).

A delta-function adjustment has been incorporated to include the forward peak contribution to multiple scattering in the techniques above. This is to account for the fact that scattering by atmospheric particulates (e.g., aerosols and cloud water droplets) is strongly peaked in the forward directions. The adjustment is achieved by appropriately rescaling the various scattering parameters to account for the assumption that radiation in the forward peak is not actually scattered at all (Wiscombe 1977; McKellar and Box 1981). Cuzzi et al. (1982) have shown that the delta-function adjustment to account for forward scattering in the case of the δ–four-stream method offers much in terms of accuracy and efficiency.

b. Computational time

As discussed in the introduction, the computational efficiency of a particular method is determined by both the accuracy and the time required to perform calculations with that method. The computational time includes the time elapsed during the execution of the program and the time spent in the system. Therefore this time will include delays due to multiuser access to CPU, so we have taken an average of numerous runs to minimize any errors involved. The results of our investigation of the computational time for the four methods are presented in Table 1. It is clear that the 32-stream method is computationally intensive, with the matrix inversion method slightly better than the δ–four-stream approximation. (The computational times include simultaneous calculation of both actinic and net fluxes for all techniques excluding the matrix inversion method, which only determines the actinic flux.) The two-stream approximation is the least computationally intensive. There is little difference in computational time for the different surface types.

c. Actinic flux and net flux

The actinic flux is defined as the radiant flux density incident onto a spherical unit area (Ruggaber et al. 1993):

$$F_{\mathrm{actinic}}(z) = \int_{4\pi} d\Omega I_\lambda(z, \Omega). \qquad (1)$$

The actinic flux considers all the radiation at a point, not just the normal component. It is an important photochemical quantity as photodissociation rate coefficients are calculated by integrating the product of the spectral actinic flux, spectral absorption cross section, and the photodissociation quantum yield over all relevant wavelengths (Madronich 1987).

In contrast, the net flux is defined as the integration of the normal component of the intensity, I_λ, over the solid angle $d\Omega$ at a given height z:

$$F_\lambda(z) = \int_{4\pi} d\Omega I_\lambda(z, \Omega)\mu, \qquad (2)$$

where $\mu = \cos\theta$ and θ is measured between the element of solid angle and the normal to the area, where the radiation is passing through.

d. Spectral integration

The model employed to determine the spectral integration is based on the correlated k distribution method (Fu and Liou 1992), which has the advantage of computational efficiency. There are 6 bands for the solar and 12 for the thermal infrared, with our calculations concentrating on the solar bands. The atmospheric model allows for gaseous absorption by water vapor, ozone, methane, nitrous oxide, and carbon dioxide. We have also investigated the first solar band (0.2–0.7 μm), which incorporates only the gaseous absorption of ozone and is, thus, pseudomonochromatic.

e. Model parameters

All aerosol types have been defined in Shettle and Fenn (1979) and reproduced in d'Almeida (1991). [See also Kay and Box (2000).]

1) Rural (added soot): This aerosol type is an external mixture of rural and soot: rural is composed of a

combination of 70% water soluble and 30% dustlike; soot consists of strongly absorbing graphitic and weakly absorbing organic particles.

2) Maritime: This aerosol consists of sea salt particles plus the background aerosol.

Note that both aerosol models are hygroscopic. Values of optical thickness and single scattering albedo as a function of relative humidity may be found in Kay and Box (2000). We have coupled these aerosol models to the appropriate surface albedo model: we have defined a terrestrial system as a rural (added soot) aerosol overlying a vegetation surface; and an oceanic system as a maritime aerosol overlying an ocean surface. The ocean surface is spectrally flat with a surface albedo of 0.05 over the six solar bands, while the vegetation surface is wavelength dependant (Stephens 1994; Kay and Box 2000). The atmospheric profile is that of midlatitude summer, except that the relative humidity (RH) profile we have used has a constant humidity up to 4 km and zero humidity above this altitude. The relative humidity in the lowest 4 km was varied between 0% and 99% during this study. Modeling assumes a plane-parallel atmosphere.

Clouds are important as they can modify the energy balance of the earth by altering the scattering (and absorption) properties of the atmosphere. Clouds are generally classified according to their position and appearance in the atmosphere. We have looked at a stratus I (ocean) cloud, (Liou 1992) with a fixed effective radius of 10.0 μm and a fixed liquid water content of 0.24 g m^{-3}. We have chosen the cloud base to be positioned at 400 m and cloud top at 800 m, that is, parameters typical for marine boundary layer clouds [stratus clouds usually have their bases below approximately 2 km; Liou (1992)].

3. Results

While calculation time has largely decreased due to significant improvements in computer technology, the focus of this paper is the parallel reduction in calculation time achieved by streamlining the calculations themselves. However the best method is seldom exclusively the fastest, nor the most accurate, but some compromise between these factors. Four different computational techniques have been compared in terms of accuracy and efficiency for varying parameters. The DISORT method is considered the most accurate but its computational speed is much slower than the other techniques, ruling it out as an efficient method in most operational situations. The accuracy of the DISORT method, however, makes it an ideal benchmark for a comparison of accuracy between the other methods.

a. Actinic flux

The first part of these results will investigate the accuracy of each of the techniques to compute actinic

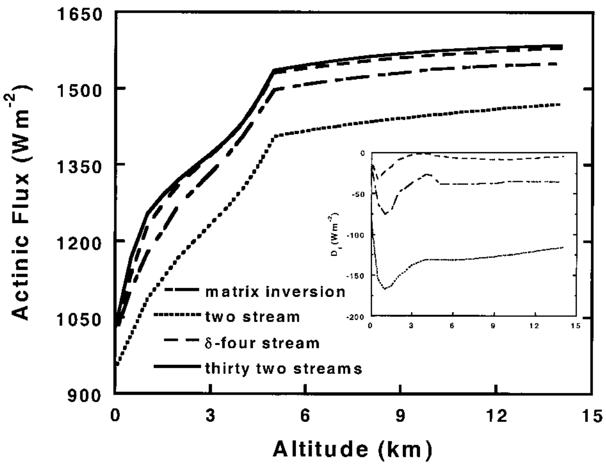

FIG. 1. Actinic flux vs altitude for the oceanic system for each computational technique at 90% RH and $\mu_s = 0.5$. The inset displays the first wavelength band, $\lambda = 0.2$–0.7 μm.

fluxes. The radiative quantities are computed as a function of solar zenith cosine, altitude, and relative humidity. Figure 1 displays actinic flux versus altitude for the oceanic system at 90% RH, for a solar zenith cosine (μ_s) of 0.5, for each of the computational techniques. [Calculations were also carried out for the first wavelength band ($\lambda = 0.2$–0.7 μm): the results are not presented here. The actinic flux values are naturally lower and the overall behavior is the same except that this band is missing the ‘‘kinks’’ found in the whole solar spectrum results in Fig. 1, due to the absence of water vapor absorption in this band.] From inspection of Fig. 1, it is clear that the two-stream approximation is the worst in terms of accuracy. The difference factor, D_f, or error in the computational techniques, can be calculated by subtracting the 32-stream result from the comparison technique that is,

$$D_f = F_{\text{computational method}} - F_{\text{32 stream method}}. \quad (3)$$

The inset in Fig. 1 displays the difference factor results. From this inset, we conclude that for the solar spectrum, the difference factor shows similar behavior for all techniques above 4 km (all techniques having a maximum difference between approximately 1 and 2 km). It is clear that the two-stream approximation is the least accurate. The difference factor for the first band is considerably smoother and also proportionally smaller than for the full solar spectrum.

Actinic flux versus solar zenith cosine for the oceanic system at an altitude of 0.5 km and 90% RH for the different techniques is shown in Fig. 2. The inset displays the difference factor for the same conditions. The matrix inversion method and δ–four-stream approximation follow roughly the same trend as the 32-stream technique, with the greatest deviation from the 32-stream approximation occurring around the middle of the solar zenith cosine range for the δ–four-stream approximation. As in Fig. 1, the two-stream approximation

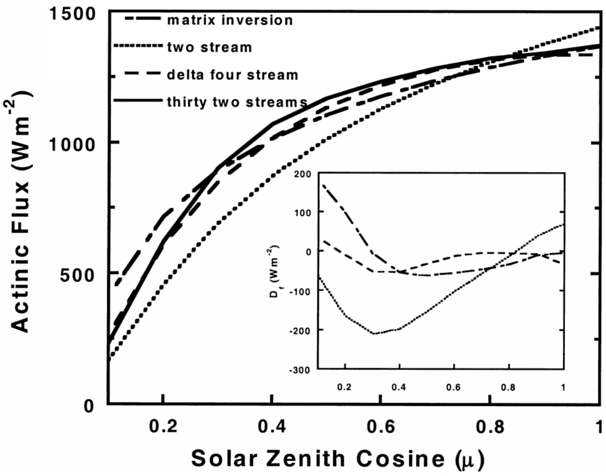

FIG. 2. Actinic flux vs solar zenith cosine for the oceanic system at an altitude of 0.5 km and 90% RH for the different computational techniques. The inset displays the difference factor D_F for the same conditions.

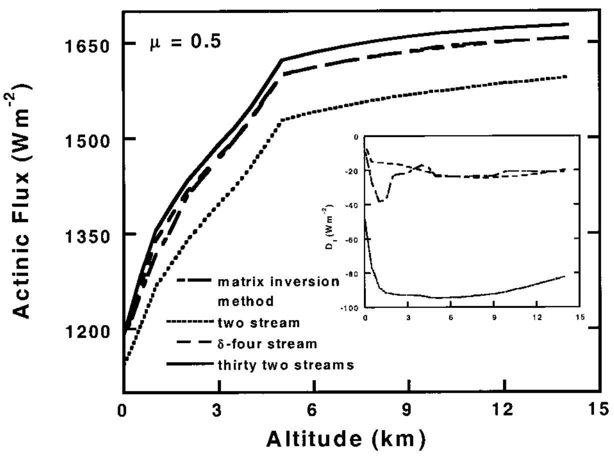

FIG. 3. Actinic flux vs altitude for the terrestrial system at 30% RH and $\mu_s = 0.5$ for each computational technique. The inset focuses on the first wavelength band $\lambda = 0.2$–0.7 μm.

deviates the most from the 32-stream version, however, the matrix inversion method deviates further at lower solar zenith cosines. This inset shows that, in general, the difference factor for the δ–four-stream approximation is uniform and relatively small compared to the other two techniques.

Figure 3 displays actinic flux versus altitude for the terrestrial system at 30% RH for each computational technique. As in Fig. 1, the main figure shows the results for the whole solar spectrum while the inset is for the difference factor. As in Fig. 1, actinic flux values above 5 km approach a constant value for all the techniques, as the relative humidity above this altitude has been set to 0%, and the aerosols are mostly found below this altitude. The 32-stream method, δ–four-stream approximation, and the matrix inversion method results all converge at ground level.

To determine the actual difference from the 32-stream method we again calculated the difference factor (Fig. 3 inset). It is clear that the error is the greatest for the two-stream approximation, while for the matrix inversion method the greatest errors occur below approximately 2 km. In the region above 3 km, there is very little difference between the δ–four-stream approximation and matrix inversion method.

The same calculations as in Fig. 2 were carried out for the terrestrial system (not shown here). For all zenith cosines, the δ–four-stream approximation was seen to be the most accurate in comparison to the 32-stream approximation. The matrix inversion method, which is not as widely known as the other techniques, generates results fairly close to the δ–four-stream approximation between solar zenith cosines of 0.4 and 1.0. (The matrix inversion method results also differ more from the δ–four-stream approximation for the oceanic system than they did for the terrestrial system.)

A plot of actinic flux against relative humidity for the terrestrial system at 0.5 km and $\mu_s = 0.5$ is shown in Fig. 4. The actinic flux decreases with relative humidity and again the two-stream approximation is the least accurate. The kink at 30% RH is somewhat artificial due to the fact that results have been computed at 0%, 30%, and then 50% RH (with increments of 10% and 5% thereafter). The difference factor is in the inset in Fig. 4, where it is now clear that the differences between the methods are relatively constant below 70% RH. Above this RH value the two-stream approximation and matrix inversion method lose a degree of accuracy, whereas the δ–four-stream approximation increases slightly in accuracy above approximately 95% RH.

The same calculations as in Fig. 4 were also carried out for the oceanic system (not shown). Once again, the actinic flux decreased with RH until around 70% RH, where it then remained relatively constant for the 32-

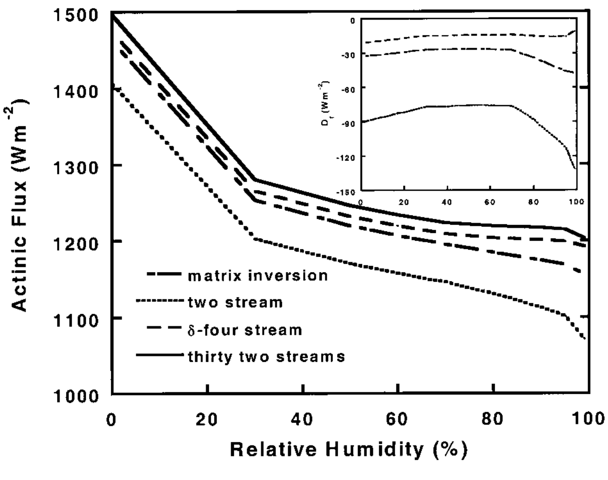

FIG. 4. Actinic flux vs relative humidity for the terrestrial system at an altitude of 0.5 km and $\mu_s = 0.5$. The inset displays the D_f for the same conditions.

stream and δ–four-stream approximations. The increase in actinic flux values for the 32-stream and δ–four-stream approximations occurs due to the hygroscopic nature of the aerosol particles (Kay and Box 2000). However computed actinic flux values slowly decreased for the other two methods. When investigating the difference factor for the oceanic system, there are similar trends to that of the terrestrial system. The two-stream approximation is the least accurate and this time the matrix inversion method is slightly better than the δ–four-stream approximation below 70% RH. The matrix inversion method and δ–four-stream approximation differ by approximately 6 W m^{-2} at 0% RH, with the difference becoming smaller until 70% RH. The optical depth of the maritime aerosol is much less than for the rural (added soot) aerosol, and also the less anisotropic the phase function is, the more accurate the matrix inversion results are. Above 70% RH, the accuracy of the two-stream approximation decreases significantly, with the matrix inversion method also decreasing in accuracy. The accuracies of both the matrix inversion method and the δ–four-stream approximation increase again above 95% RH.

In comparison to the error values for the terrestrial system, the deviations from the ''benchmark'' 32-stream method are greater for the oceanic system. This is most likely due to the fact that the oceanic system is cleaner (less absorbing) and is more affected by changes in relative humidity, so that the effects of growth become important. The differences between the methods at the different relative humidity values are more due to the way in which each program handles the strongly peaked phase function of large aerosol particles. As a final point, it should be noted that the difference factor results and Figs. 1 and 3 confirm that different parameters and model inputs affect the accuracy of each approximation, demonstrating that care is needed when choosing an appropriate method to use for a particular system.

b. Flux computations

All of the above analyses were repeated to obtain the net flux, for both surfaces and aerosol types used in the previous subsection. This time only three of the methods have been compared (32 stream, δ–four stream, and two stream) as the matrix method does not calculate the net flux. In this section we will mainly focus on the flux at the surface.

Figure 5a displays the surface net flux as a function of relative humidity for both cases—the terrestrial and oceanic systems—for a solar zenith cosine of $\mu_s = 0.5$. The difference factor was computed for both systems and is shown in Fig. 5b. From Fig. 5a it is clear that there are similar features in the results for the two surface–aerosol combinations. The two-stream approximation is clearly the less accurate, with an almost negligible difference between the δ–four-stream approximation and 32-stream methods except at the highest RH values. We did notice, however, that at lower μ_s values (~0.2), the deviation between the δ–four-stream and 32-stream methods increases slightly in comparison to higher μ_s values. The difference factor for both approximate methods increases between 70% and 99% RH; however, this increase is more significant in the case of the two-stream approximation, where an increase of approximately 5–6 W m^{-2} was observed. The reason for this increase is aerosol growth (Kay and Box 2000), and it is clear that the effects of growth become important at high RH and increase the errors with increasing RH as observed in Fig. 5b. Note that this increase is more significant for the oceanic system. The reason for this is that the maritime aerosol has a higher water-soluble content and hence the effects of growth are more pronounced for this system.

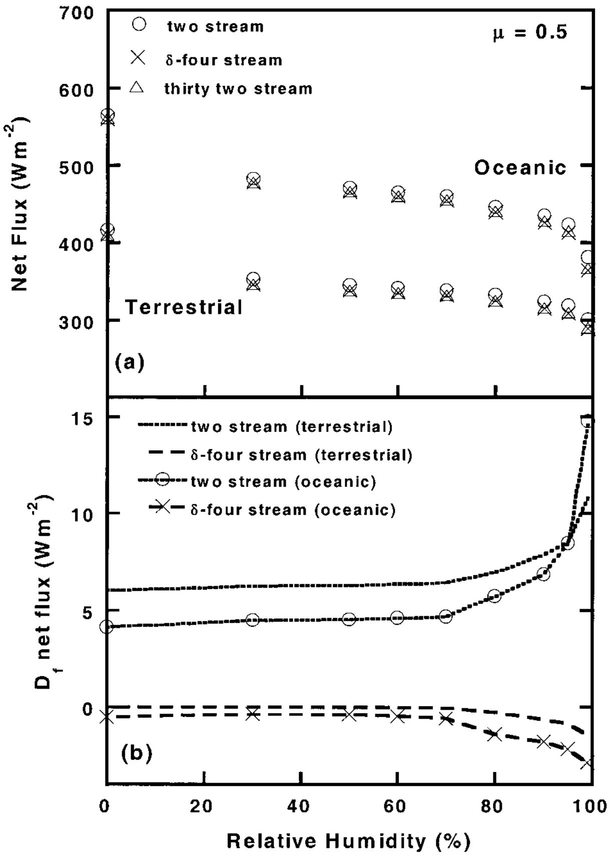

FIG. 5. (a) Surface net flux vs relative humidity at $\mu_s = 0.5$ for the terrestrial and oceanic systems. (b) Difference factor for the same conditions as in (a).

Figure 6 displays the net flux for the terrestrial system versus altitude at 30% RH, and the inset shows the difference factor. The relatively constant results above 5 km are again mainly due to the absence of water vapor above this altitude. There is a steady increase in flux values until 5 km, with only a slight deviation in all the methods (the most obvious is again the two-stream approximation) as is confirmed by the difference factor.

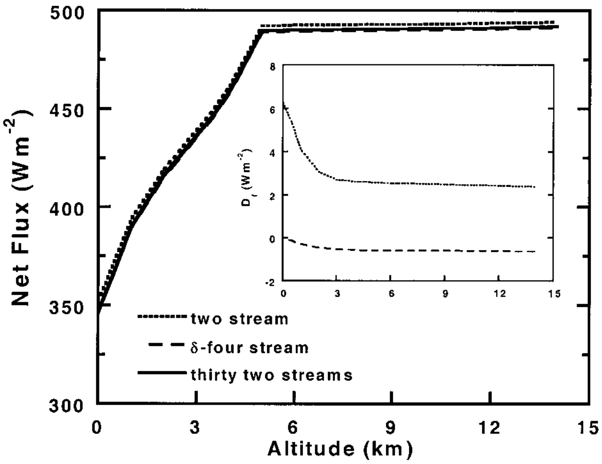

FIG. 6. Net flux for the terrestrial system vs altitude at 30% RH and μ_s = 0.5 for the computational techniques. Inset in (a) is net flux for the first band, and in (b) the difference factor for the same conditions.

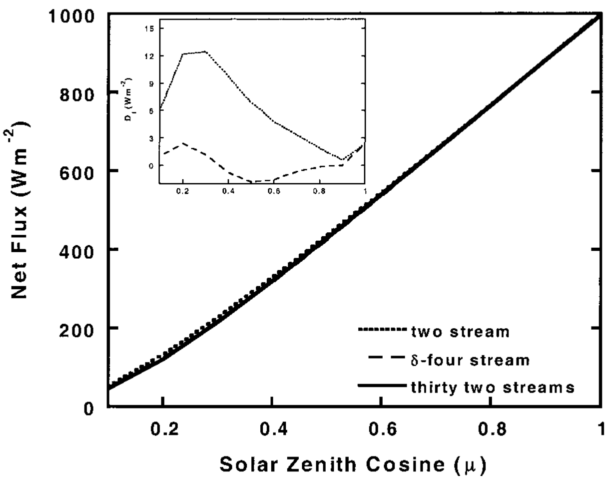

FIG. 7. Net flux at the ground as a function of solar zenith cosine for the oceanic system at 90% RH for the computational techniques. The inset displays the difference factor for the same conditions.

The difference factor shows that the greatest error occurs below 3 km, which is where the bulk of the aerosol layer is located. The main difference between the first band (not presented here) and the whole spectrum lies in the magnitude of the net flux values, and the absence of the kink at the top of the water vapor layer.

The net flux at the ground, for the different methods as a function solar zenith cosine for the oceanic system for 90% RH, is plotted in Fig. 7. At lower solar zenith cosines, the two-stream approximation differs slightly from the other methods. This is clearer in Fig. 7 (see the inset) where there is a maximum deviation at approximately $\mu_s \approx 0.3$ of nearly 12 W m^{-2}. The δ–four-stream approximation oscillates within 3 W m^{-2}, indicating that in terms of overall accuracy for the net flux at the ground, the δ–four-stream approximation is superior to the two-stream approximation.

c. Clouds

As in the previous sections, we have repeated our investigations of the comparison of the different methods, this time focusing on the effects clouds have on calculations of both actinic and net fluxes. Figure 8 displays actinic flux for an ocean surface and maritime aerosol at 99% RH for a stratus (I) cloud for μ_s = 0.5. We have concentrated on the differences between the methods for the lowest kilometer of the atmosphere. This includes the cloud layer between 400 and 800 m, with a vertical resolution of 25 computational layers in the lowest 1 km. (Other resolution values have been investigated, with the conclusion that a greater number of layers produces closer results between the different methods. The impact of vertical resolution on accuracy is under further investigation, and the results will be published elsewhere.) From Fig. 8 it is clear that the matrix inversion method deviates quite substantially both above and below the cloud range. On inspection of the inset in Fig. 8, it is clear that the two-stream approximation, and especially the matrix inversion method, have problems within the cloud but more substantially above the cloud layer. The assumption of isotropic scattering is mainly responsible for the poorer results for the matrix inversion method and two-stream approximation for calculations that include clouds, whereas the δ–four-stream approximation can handle these calculations with relative ease. In comparison with Fig. 1 (for a cloud-free atmosphere), actinic flux values slowly increase, whereas with a cloud present the values remain almost constant below the cloud. There is also a larger actinic flux range with a cloud present, due to the higher scattering in the cloud. This is consistent with the work of Los et al. (1997) who also noted that in

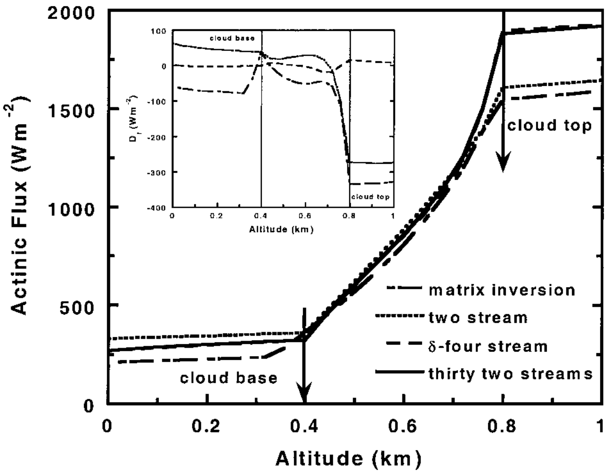

FIG. 8. An ocean surface and maritime aerosol at 99% RH and μ_s = 0.5 for a stratus (I) cloud for actinic flux. The insert is the difference factor.

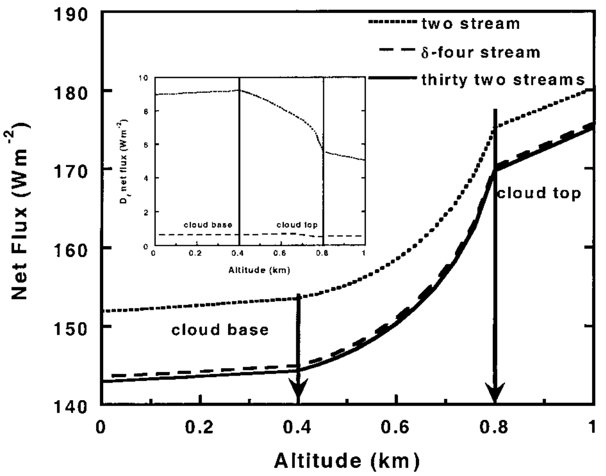

FIG. 9. An ocean surface and maritime aerosol at 99% RH and $\mu_s = 0.5$ for a stratus (I) cloud net flux. The insert is the difference factor.

cloudy conditions there were considerable changes in actinic fluxes.

Figure 9 is for the same conditions as Fig. 8, this time considering net rather than actinic flux. The main difference to note is that the range of net flux values within the cloud is much smaller than the range found for actinic flux. The difference factor has also been calculated and is displayed in the inset to Fig. 9. The δ–four-stream approximation has very small difference values in comparison to the two-stream approximation. The accuracy of the two-stream approximation, however, increases within and above the cloud. If both of the insets are compared, it is interesting to note that the difference errors are far greater for actinic flux calculations for the two-stream approximation and matrix inversion method, highlighting the effect of the impact a cloud layer has on the accuracy of these techniques.

4. Summary and discussion

This section summarizes the difference factor results, that is, the relative accuracy of each computational technique. The first situation we will investigate is difference factor versus relative humidity. Table 2 summarizes actinic flux calculations for the terrestrial system, with absolute and percentage errors for each technique. Below 70% RH, the factor is relatively constant for all three methods and both surface–aerosol combinations. The matrix inversion method had greater errors at the higher RH values, while the δ–four-stream approximation had smaller errors occurring at the higher RH values. The absolute errors for the oceanic system were larger by approximately 10–20 W m^{-2} for both the δ–four-stream approximation and matrix inversion method (percentage errors for the δ–four-stream approximation approximately doubled compared to the terrestrial system; however, the matrix inversion method percentage errors stayed approximately the same). For the two-stream approximation both absolute and percentage errors were larger than the terrestrial system (errors were in the range of 112–189 W m^{-2}, or approximately 8%–17%).

For the case of net flux against relative humidity, for both surface–aerosol combinations the difference factor is constant below 70% RH. In both cases the δ–four-stream approximation is the most accurate. Above 70% RH the difference factor of both methods decreases, for both surface–aerosol combinations. The δ–four-stream approximation errors were negligible (less than 1%) while the two-stream approximation for the oceanic system had errors in the range of 4–14 W m^{-2} (~1%–4%). For the cases of net flux against altitude, we saw the same trends for both surface–aerosol combinations. Errors are relatively constant for each technique above 3 km; however, the two-stream approximation increases in accuracy with altitude. The δ–four-stream approximation has almost negligible errors. Investigations were also carried out for a standard atmosphere (midlatitude summer) and the general trends and errors are comparable.

Comparing the accuracy and time with a cloud present required taking note of the fact that the computational time was increased as the layering in the lowest kilometer had to be increased (see Table 1). For the cases when a cloud is included, particularly interesting results occurred. The main points to note and the errors involved are the δ–four-stream approximation, for calculations of net and actinic fluxes, has negligible errors as shown in Figs. 8 and 9. This approximation handles calculations within clouds very well. As is seen in Table

TABLE 2. Absolute and percentage errors for actinic and net flux for the three comparison techniques.

Computational technique	Actinic flux: Absolute error (W m^{-2})	Actinic flux: Percentage error (%)	Net flux: Absolute error (W m^{-2})	Net flux: Percentage error (%)
Two-stream approximation	90–131	~6–11	6–10	~1–4
Matrix inversion method	30–47	~2–4	Not available	Not available
δ–four-stream approximation	10–22	~1–2	Negligible	<1

TABLE 3. Absolute and percentage errors for actinic and net fluxes for the three comparison techniques with a cloud present.

Computational technique	Actinic flux: Absolute error (W m^{-2})	Actinic flux: Percentage error (%)	Net flux: Absolute error (W m^{-2})	Net flux: Percentage error (%)
Two-stream approximation	+62 to −273	Max ∼20	5–9	3–6
Matrix inversion method	+60 to −328	Max ∼25	Not available	Not available
δ–four-stream approximation	Negligible	Negligible	Negligible	Negligible

3, the matrix inversion method had a maximum error of approximately 25%, with minimum errors below and above the cloud base for actinic flux. Note that in this case, the two-stream approximation is significantly less accurate when computing actinic as opposed to net fluxes.

For the case of actinic flux, it is clear that the accuracy of the δ–four-stream approximation is superior to that of the other two techniques. Inclusion of a cloud for the cases of net flux had very little effect on the accuracy of the δ–four-stream approximation (see Fig. 9).

5. Conclusions

In this paper we have investigated the accuracy and efficiency of four relatively well-known and documented computational techniques—DISORT (run with 32 streams), the δ–four-stream approximation, a version of the two-stream approximation, and the matrix inversion method—for application to broadband calculations. All techniques were examined over a range of parameters: varying solar zenith cosine, relative humidity, and altitude, for two different surface–aerosol combinations, for the determination of actinic flux values. Calculations were also repeated for all techniques excluding the matrix inversion method for the determination of net flux values. Further investigations were also made into the difference between the four techniques for inclusion of a cloud layer in the oceanic system. In this paper we have attempted to provide information to assist users in choosing a computational technique that would best suit their needs.

In this paper, we have tried to provide readers with a range of parameters whereby a decision could be made as to which technique might best be applied to their work. The difference term was examined to quantify the errors for each technique as a function of the parameters relative humidity and altitude. We found that the two surface–aerosol combinations led to different levels of accuracy for the cases against relative humidity. The calculations made with the terrestrial system were slightly more accurate than the oceanic system, with the δ–four-stream approximation as the most accurate of all the techniques for the terrestrial system. For an oceanic system the matrix inversion method was slightly more accurate than the δ–four-stream approximation.

After consideration of all results obtained, we have determined that, in terms of accuracy, the δ–four-stream approximation is slightly better than the matrix inversion method without clouds, and both are clearly better than the two-stream approximation. However, with clouds included, the δ–four-stream approximation is clearly more accurate than the other techniques. Therefore, overall, the technique that stands out in terms of accuracy and efficiency is the δ–four-stream approximation for both net and actinic flux calculations.

When taking into account the computational time, one must note that the quoted computational times are for the simultaneous calculation of both actinic and net fluxes for three of the methods, while for the matrix inversion method they are for the calculation of actinic flux only. All the information we have gathered from our investigations is of course only valid for the particular versions of the techniques that we have used and the choice of which technique is the superior one may also be determined by the level of accuracy the reader is looking for, what conditions they choose to employ, and the time available to perform the computations. Therefore, unless time is the primary concern we recommend the δ–four-stream approximation as our preferred technique, especially in the case of actinic fluxes.

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