Empirical Formulation of a Debye-Layer Property of Nuclei

H. Hora
Department of Theoretical Physics, University of New South Wales, Sydney 2052, Australia
h.hora@unsw.edu.au

The formulation of the Debye-length for electrons from plasma physics was generalized to the degenerate electrons of metals by changing the temperature to Fermi energy. Going further to take the nucleons and their Fermi energy, we show here that a Debye length appears of few 10^{-13} cm which reproduces Hofstadter's few fm decay [1,2] for the nucleon density of nuclei. This may be interpreted for a drop model [3] of nuclei as unsaturated bonds of the 2-3 fm Yukawa potential at the surface of nuclei resulting in a surface energy for confining the nucleons in a nucleus. Relations for the relativistic change of the Fermi energy of the nucleons around and higher than nuclear density may indicate the quark-gluon state.

The Debye length

$$\lambda_d = (kT/4\pi e^2 n_e)^{1/2} \qquad (1)$$

in classical plasmas of a temperature T and electron density n_e as result of the Debye-Milner theory [4] can be derived as the thickness of the electric double layer in the surface of a uniform plasma expanding against vacuum e.g. after being generated by laser irradiation (see Fig. 2.2 p, 31 of Ref. 5 or Fg. 2-2, p. 13 of Ref. 6). There the electrons at thermal equilibrium are leaving the surface due to their smaller mass than that of ions until a one dimensional surface potential of kT/2 is built up returning further electrons from leaving the plasma. The electric field energy in this layer per surface area results in a double a layer [5,6] causing a surface tension [5,7,8]

$$\sigma_e = 0.27(kT)^2/8\pi e^2\lambda_d) \qquad (2)$$

This surface energy is too small to confine laser produced plasmas, but the stabilization of surface waves up to wave length of about 80 Debye length [8] against the Rayleigh-Taylor instability is sufficient to produce always smooth plums of expanding laser produced plasmas.

The same happens in a metal between the ions where the then degenerate electrons like to leave the lattice. In this case kT has to be exchanged against the Fermi energy of the electrons in Eqs. (1) and (2). This results in the well known work function of the metals as an exit potential of about 3 to 10 eV and in a surface energy (tension) as measured and evaluated by R.S. Pease et al [9] where minor corrections are due to the effective electron masses [5].

The next step of generalization considers the nucleons (protons and neutrons) of nuclei instead of electrons and uses the subrelativistic Fermi energy

$$E_F = (3n_N/\pi)^{2/3}h^2/(8m_N) \qquad (3)$$

of the nucleons with a mass m_N and density n_N. The nucleon Debye length is then

$$\lambda_N = [E_F/(4\pi e^2 n_N)]^{1/2} \propto n_N^{-1/6} \qquad (4)$$

Taking the experimentally known values of nucleon densities, scattering very closely around the nucleon density in a nucleus

$$n_N = 8.5x10^{37} \text{ cm}^{-3} \qquad (5)$$

resulting in a Fermi energy for nucleons of 36 MeV, we find a Debye length of

$$\lambda_N = 3.4x10^{-13} \text{ cm} \qquad (6)$$

This length corresponds quite well with Hofstadter's [1,2] measured density decay of about 3 fm of nucleons at the surface of heavy nuclei down to about calcium (see Fig. 14b of Ref. 2) in rather good agreement with the nucleon Debye length (6). One may not consider this fact as a coincidence but as a result of the just given steps for the generalization of the Debye length for the conditions of the nucleons in a nucleus.

After this basic and well convincing result we may consider the fact from Eq. (4) to understand why the 3fm-Hofstadter decay is so highly independent on the compared value of the nucleon Debye length (4). The nucleon density in the inner part of the nuclei is nearly the same for all nuclei between vanadium and bismuth which fact was involved in the average value of Eq. (5). These nuclei show then a rather unchanged decay profile due to the then unchanged Debye length (see Fig. 14b of Ref. 2). An indication of a Debye layer mechanisms can be seen when comparing the measurements of Fig. 1 for calcium with the lower central nucleon density than that of gold. While the ratio of the central densities is 1.060, the ratio of the Debye lengths following Eq. (4) is 0.990 and an evaluation from the points in Fig.1 using the original values to minimize the error bars is 0.995 in acceptable agreement.

After this semi-empirical relation, we consider the confining of a number Z of protons and a number A-Z of neutrons in a nucleus. The primary repulsion is by the Coulomb force until a diameter larger than about 10 times the known nuclear diameters, but then the main resistance against further squeezing is the Fermi energy growing with another exponent than the Coulomb energy. The Fermi energy for the nucleons is [Eq. (1.45, Ref. 10]

$$E_F = [3/(\pi)^{2/3}/4][h^2 n^{2/3}/(2m)](\lambda_C/2)^{-1}[n+1/(\lambda_C/2)^3]^{-1/3} \qquad (7)$$

Where λ_C is the Compton wave length of the nucleons defining the subrelativistic branch for nucleon distance larger than λ_C. The surface energy E_{surf} of A nucleons with a Debye length λ_d and a Fermi energy E_F of the nucleons is [11]

$$E_{surf} = 0.27[3A(4\pi)^{1/2}]^{2/3}3^{1/3}E_F^{2/3}/(\pi^{1/2}2^{5/2}n^{1/6}e) \qquad (8)$$

Taking the total Fermi energy of the nucleons in a nucleus as the inner energy with a negligible contribution of the Coulomb energy, the surface energy is always smaller than the surface energy for lower densities until the nucleon density $n=n_N$ reaches about the measured value, Eq. (5). This may be interpreted that then the confining of the nucleons is due to a range of the Debye depth at the nuclear surface as if the Yukawa potential of comparable length at the nuclear surface causes unsaturated bonds which are compensated by the Debye layer. This relates to the matter wave propagation at particle scattering into the lower density area at total reflection of matter waves as considered by Wigner [12] and clearly confirmed as an optical Goos-Haenchen effect [13] with time resolution of femto- to attoseconds [14] based on the special second order difference of the matter waves against optical waves [15,16]. While the temporal Goos-Haenchen effect in the optical case [13] shows the clear difference for contrasting polarisation, the Wigner [12] consideration indicates a problem which was evaluated for electron waves [15,16] that there is a drastic and yet unsolved difference between a phase (Ψ-treatment) against an intensity ($\Psi^*\Psi$-treatment). A further important connection is to the optical cases going into the nonlinear conditions [6,17].

If the density n_N of the nuclei is more than about 10 times higher than the measured nuclear density, the relativistic branch in Eq. (7) is valid and the ratio of surface energy to the inner energy of the nucleons is

$$E_{surf}/(AE_F) = 0.27[3^{8/3}/(2^{7/3}\alpha^{1/2}A^{1/2})] \qquad (6)$$

This is then always less than unity whatever value A is used where α is the fine structure constant. Since this ratio does not depend - as typical relativistic expression – neither on the particle mass m nor on their density n, it is not defined whether the masses are neutrons of neutron stars or whether this soup of matter without any possible nucleation consists of quarks and gluons or other particles of any mass. If this matter of higher than nuclear density in the early state of the big pang is expanding, it will then reach the subrelativistic branch of the Fermi energy of nucleons at the known nuclear density in which state the nucleation would occur [18]. Properties of Debye lengths are well known in quark-gluon plasmas [19].

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Fig. 1 Nucleon density in Ca and Au nuclei [2] with the decay at the surface using the points for comparison with the nucleon Debye length.