Nuclear Force

Nuclear Force

What keeps the nucleus whole ?

H atom: Coulomb, EM force
Nucleus: protons have q > 0
neutrons have q = 0
so it can't be EM.
Gravitational ? Far too weak !!

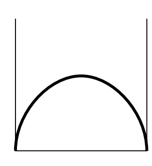
$$\frac{F_g}{F_e} \approx 10^{-36}$$

Nuclear Force

- The nuclear force is very strong (the nuclear potential is deep)
- Short-range: proton scattering – if distance from nucleus is $\geq$ 2-3 fm, the proton only 'sees' the Coulomb (EM) force.
- The range of the nuclear force is < 2 fm
- The strong nuclear force between nucleons is ~ independent of charge
  n – n, n – p, p – p
- The strong nuclear force is spin-dependent (see deuteron).
- Known as "The Strong Force"
- Not all particles experience the Strong Force e.g. electrons

Strength of nuclear force

Imagine a nucleon moving inside a cubic potential box of side length 5 fm.



L

$$\lambda = 2L/n$$

$$p = \frac{h}{\lambda} = \frac{nh}{2L}$$

$$E = \frac{p^2}{2m} = \frac{n^2 h^2}{8mL^2}$$

$$E_{3D} = \frac{\left(n_x^2 + n_y^2 + n_z^2\right) h^2}{8mL^2}$$

- The ground state has

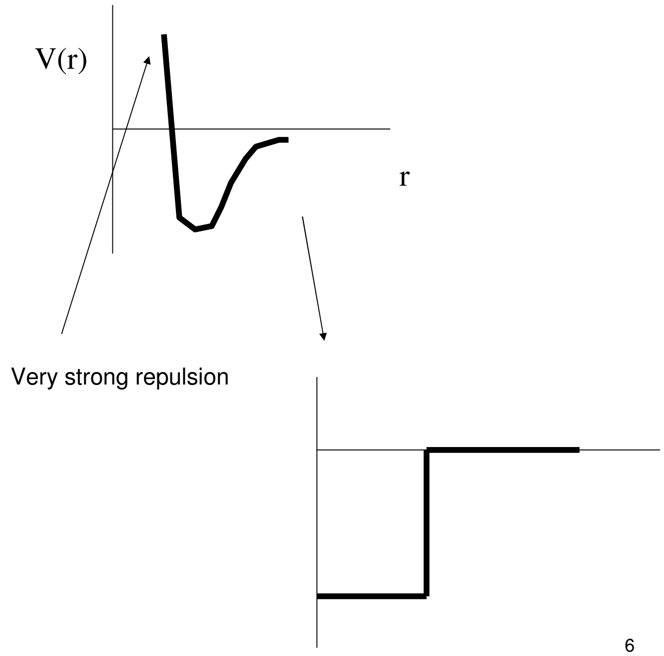
$$n_x = n_y = n_z = 1$$

$$E = 3h^2/8mL^2 \approx 25 \ MeV$$

cf electron in H atom –13.6 eV

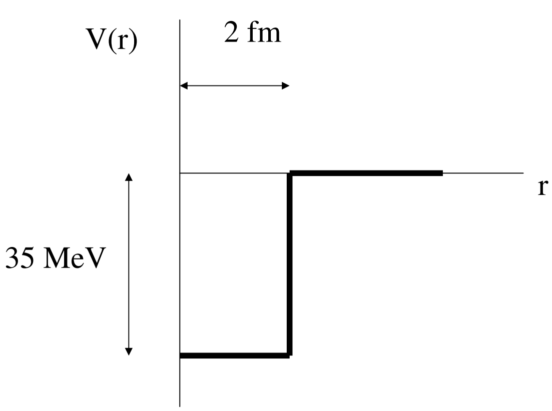
Potential

- Spherical, square-well potential



Nuclear forces

- Deuteron shows there is a strong, spin-dependent force between the proton and the neutron



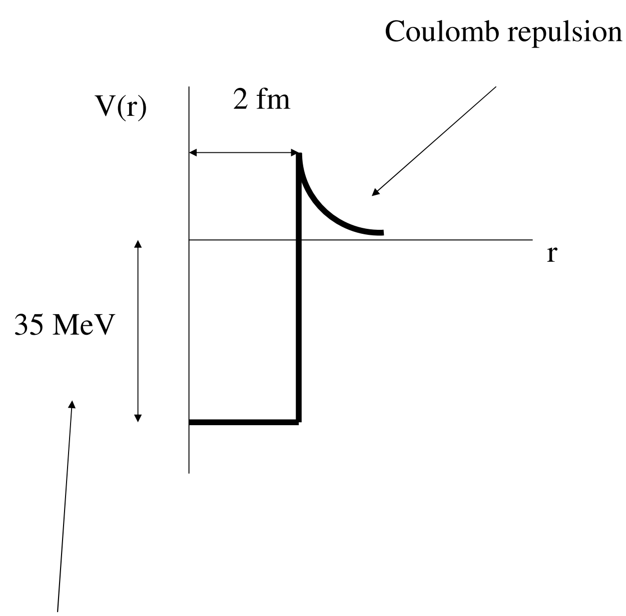
Neutron scattering off protons (or neutrons)

Very short-range force

Charge-independent

Nuclear forces

- Proton-proton scattering



Same as for n-n and n-p scattering

The Deuteron (d)

- Simplest nucleus with more than one nucleon

- 1 proton + 1 neutron
- Charge = +1, Mass ~ 2 u
- m_p = 1.007276 u
- m_n = 1.008665 u
- $m_p + m_n$ = 2.015941 u
- m_d = 2.013553 u

$$m_d < m_p + m_n$$

The system is BOUND

Deuteron

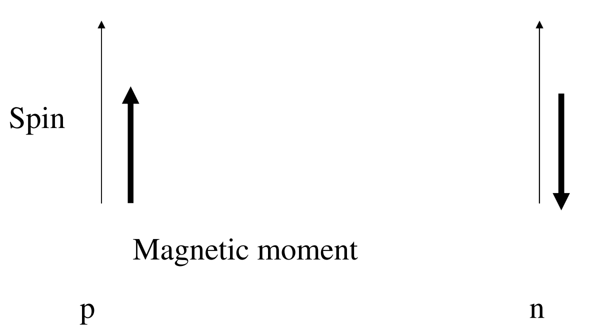
- Binding Energy = ?

$$\Delta m = 2.015941 - 2.013553\ u$$
$$= 0.002388\ u$$
$$= 0.002388 \times 1.66 \times 10^{-27}\ kg$$
$$= 0.002388 \times 1.66 \times 10^{-27} \times \left(3.0 \times 10^{8}\right)^{2} J$$
$$= 3.57 \times 10^{-13} J = 2.23 MeV$$

Large energy indicates a strong attractive force between the proton and the neutron

Deuteron

- Magnetic moment = +0.8574 μ_N
- Proton = +2.7928 μ_N
- Neutron = –1.9130 μ_N



$$\mu_d = |\mu_p| - |\mu_n| = +0.8798 \mu_N$$

Deuteron

- Nuclear spin (expt): $I^{\pi} = 1^{+}$

$$\vec{I} = \vec{s}_p + \vec{s}_n + \vec{l}$$

$$l_n = 0$$

Options

$l = 0 \quad \& \quad \uparrow\uparrow$

$l = 1 \quad \& \quad \uparrow\uparrow$

$l = 1 \quad \& \quad \uparrow\downarrow$

$l = 2 \quad \& \quad \downarrow\downarrow$

Parity = +1 $\quad \pi = (-1)^{l} = +1$

$$\therefore \quad l = 0, 2$$

Spin-dependence

- d = p + n

$$S_p = \frac{1}{2} \qquad S_n = \frac{1}{2}$$

$$\vec{S}_d = \vec{S}_p + \vec{S}_n$$

- possibilities are

$$S_d = 0 \quad (\uparrow\downarrow) \qquad S_d = 1 \quad (\uparrow\uparrow)$$

- Experimentally, the deuteron has only 1 bound state with $S_d = 1$
- Therefore, the strong interaction is spin-dependent !

Deuteron

- Magnetic moment

$$l = 0 \quad \rightarrow \quad \mu_{orb} = 0$$

$$\vec{\mu}_d = \vec{\mu}_p + \vec{\mu}_n = 0.8798\mu_N$$

$$\mu_{\text{expt}} = 0.8574\mu_N$$

$$\psi = a_s\psi_s + a_d\psi_d$$

$l = 0$ state → ψ_s; $l = 2$ state → ψ_d

$$a_s^2 \approx 0.96$$

$$a_d^2 \approx 0.04$$

Deuteron

- Electric Quadrupole moment

$$Q_p = Q_n = 0$$

$$Q_d \neq 0 \quad (0.00288\, b)$$

- Further evidence for admixture of the $l = 0$ and $l = 2$ states.
- Shows that the interaction potential must have a non-central (tensor) component.

Deuteron

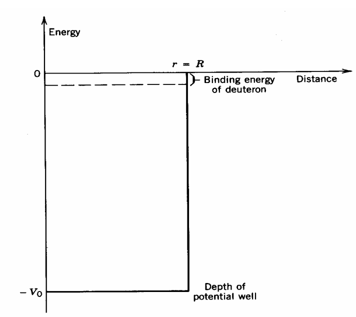
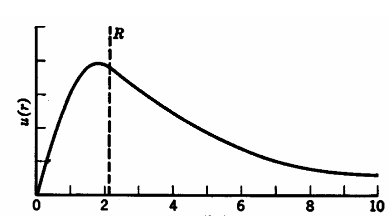


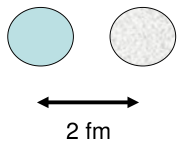
Figure 4.1 The spherical square-well potential is an approximation to the nuclear potential. The depth is $-V_0$, where V_0 is deduced to be about 35 MeV. The bound state of the deuteron, at an energy of about -2 MeV, is very close to the top of the well.



Krane 4.1 & 4.2

Nucleon-nucleon scattering

Deuteron – only one bound configuration



$l = 0$ & $\uparrow\uparrow$

n-n, n-p and p-p scattering shows that the interaction potential is "Charge symmetric" once you correct for p-p Coulomb interaction

NOT electric charge ------ nucleon p or n

Interaction potential

First term

Central potential i.e. no θ, φ dependence

$$V_C(r)$$

Second term

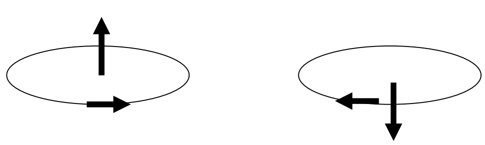
Add a term to account for the fact that

$$E(\uparrow\uparrow) \neq E(\uparrow\downarrow)$$

Interaction potential

The interaction between nucleons is invariant under parity and time-reversal

$$\vec{r} \to -\vec{r} \quad \& \quad t \to -t$$



Spin-dependent terms cannot be linear in spin

$$\vec{S} = \vec{s}_1 + \vec{s}_2$$

$$\vec{S}\cdot\vec{S} = (\vec{s}_1 + \vec{s}_2)\cdot(\vec{s}_1 + \vec{s}_2)$$

$$= \vec{s}_1\cdot\vec{s}_1 + \vec{s}_2\cdot\vec{s}_2 + 2\vec{s}_1\cdot\vec{s}_2$$

$$\therefore \quad \vec{s}_1\cdot\vec{s}_2 = \tfrac{1}{2}[\vec{S}\cdot\vec{S} - \vec{s}_1\cdot\vec{s}_1 - \vec{s}_2\cdot\vec{s}_2]$$

$$= \tfrac{1}{2}[S(S+1) - s_1(s_1+1) - s_2(s_2+1)]$$

$$\langle \vec{s}_1\cdot\vec{s}_2 \rangle = \frac{\hbar^2}{2}[S(S+1) - s_1(s_1+1) - s_2(s_2+1)]$$

Interaction potential

Triplet state: S = 1 $\uparrow\uparrow$ (2S+1) = 3

$$\langle \vec{s}_1\cdot\vec{s}_2 \rangle = \frac{\hbar^2}{2}[1(1+1) - \tfrac{1}{2}(\tfrac{1}{2}+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)] = \frac{\hbar^2}{4}$$

Singlet state: S = 0 $\uparrow\downarrow$ (2S+1) = 1

$$\langle \vec{s}_1\cdot\vec{s}_2 \rangle = \frac{\hbar^2}{2}[0(0+1) - \tfrac{1}{2}(\tfrac{1}{2}+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)] = \frac{-3\hbar^2}{4}$$

$$E(\uparrow\uparrow) \neq E(\uparrow\downarrow)$$

Interaction potential

Third term: deuteron has a Quadrupole Moment

NOT a pure $l = 0$ state – a small admixture of the $l = 2$ state

So, there is a non-central, 'Tensor' component

Take the spin as the reference direction

$$\vec{s}\cdot\vec{r} \quad \text{or} \quad \vec{s}\times\vec{r}$$

$$(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r}) \quad \text{or} \quad (\vec{s}_1\times\vec{r})\cdot(\vec{s}_2\times\vec{r})$$

Can be written in terms of

$$(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r}) \quad \& \quad \vec{s}_1\cdot\vec{s}_2$$

Interaction potential

Write Tensor term as:

$$C(r)\,S_{12} = C(r)\left\{\frac{3(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r})}{r^2} - \vec{s}_1\cdot\vec{s}_2\right\}$$

Vanishes for Singlet state

$$\vec{s}_1 = -\vec{s}_2$$

Therefore, the interaction potential can be written as:

$$A(r) + B(r)\,\vec{s}_1\cdot\vec{s}_2$$
$$+C(r)\left\{\frac{3(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r})}{r^2} - \vec{s}_1\cdot\vec{s}_2\right\}$$

Nuclear STRONG force Particle exchange

- Yukawa (1935, Nobel 1949)
- Analogy with EM and covalent bonding (e.g. H_2 molecule).
- EM $\quad q_1 \rightarrow \vec{E} \qquad \vec{F}_2 = q_2\vec{E}$
- t-dependent changes in q_1 distribution → t-dep changes in **E** field i.e EM radiation – light – photons

$$\Delta E\,\Delta t > \hbar$$

$$\Delta E = hf \;\therefore \Delta t > 1/2\pi f$$

$$c\Delta t = \lambda/2\pi$$

Nuclear STRONG force Particle exchange

- EM : 2 charges swap virtual photons
- Range of interaction:

$$R = c\Delta t = \lambda/2\pi$$

- m = 0 so no restriction on λ
- R → ∞
- Yukawa – the nuclear strong force has a finite range ≤ 2 fm
- Assume nucleons exchange a particle with m ≠ 0

Nuclear STRONG force Particle exchange

Estimate the mass of the exchange particle

$$\Delta E \geq mc^2$$

$$R < c\Delta t$$

$$\therefore R < \frac{\hbar}{mc}$$

$$\therefore m < \frac{\hbar}{Rc} = \frac{1.05\times10^{-34}}{2\times10^{-15}\times3\times10^{8}}$$

$$= 1.75\times10^{-28}\ kg$$

$$\therefore mc^2 = 1.6\times10^{-11}\ J = 100\,MeV$$

Nuclear STRONG force Particle exchange

A nucleon continually emits and absorbs virtual mesons

Estimate the lifetime

$$\Delta t \approx \frac{\hbar}{mc^2}$$

$$mc^2 = 140\,MeV = 140\times10^{6}\times1.6\times10^{-19}\ J$$

$$\therefore \Delta t \approx \frac{1.05\times10^{-34}}{140\times10^{6}\times1.6\times10^{-19}}$$

$$\approx 5\times10^{-24}\ s$$

hence VIRTUAL

Nuclear STRONG force Particle exchange

- Yukawa calculated the lifetime of this exchange particle whose mass is of order 100 MeV/c^2
- lifetime ~ 1 μs
- Particle referred to as a "meson" ('middle' – mass is intermediate between electron and proton)
- 1936 Anderson & Neddermeyer discovered the Muon (cosmic rays)

$$m(\mu^+) = m(\mu^-) = 106\ MeV/c^2 \quad (207 m_e)$$

$$\tau_\mu \sim 2.2\,\mu s$$

- "Mu-meson" (wrong !)

Nuclear STRONG force Particle exchange

- Yukawa's meson lifetime calculation had an error – factor of 100 too big !
- lifetime ~ 10 ns
- Muons are not attracted strongly to nucleus !
- Muon can spend time inside nucleus without being absorbed
- Muon has $s = \frac{1}{2}$ so muon exchange between nucleons won't conserve angular momentum
- We now know that the muon is elementary – it's NOT a Meson (later).

Nuclear STRONG force
Particle exchange

- 1947 – Powell et al. discovery of Pions (pi-mesons)

$$m(\pi^+) = m(\pi^-) = 140\ MeV/c^2 \quad (274 m_e)$$

$$\tau_\pi \sim 26\, ns$$

- 1950 – Moyer et al.

$$m(\pi^o) = 135\ MeV/c^2 \quad (264 m_e)$$

- Pions have $s = 0$ so pion exchange between nucleons will conserve angular momentum

Nuclear STRONG force
Particle exchange

- Pion exchange mechanisms

$$n \to n + \pi^o \quad \Leftrightarrow \quad \pi^o + p \to p$$

$$n \to p + \pi^- \quad \Leftrightarrow \quad \pi^- + p \to n$$

$$p \to n + \pi^+ \quad \Leftrightarrow \quad \pi^+ + n \to p$$

- Nucleons swap identities in about 50% of the events
- Pion exchange could also provide an 'explanation' for the magnetic moment of the uncharged neutron

$$n \to p + \pi^-$$

Both charged

Yukawa potential

- Consider the electrostatic potential around a point charge
- $\nabla^2 \phi = 0$ Laplace
- Solution is

$$\phi = \frac{e}{4\pi\varepsilon_0 r}$$

- This describes a force mediated by massless particles - photons

Yukawa potential

- Particles with mass
- Klein-Gordan equation (1926)

$$E^2 = (pc)^2 + (mc^2)^2$$

$$E \to i\hbar \frac{\partial}{\partial t} \qquad\qquad p \to -i\hbar\nabla$$

$$\nabla^2 \psi(r,t) - \frac{1}{c^2}\frac{\partial^2 \psi(r,t)}{\partial t^2} = \left(\frac{mc}{\hbar}\right)^2 \psi(r,t)$$

- Important in QED – photons m = 0

$$\nabla^2 \psi(r,t) = \frac{1}{c^2}\frac{\partial^2 \psi(r,t)}{\partial t^2}$$

- Classical wave equation

Yukawa potential

- Now consider pions in a 'static', time-independent case:

$$m_\pi \neq 0$$

$$\left(\nabla^2 - \left\{\frac{mc}{\hbar}\right\}^2\right)\Phi(r) = 0$$

- Spherical symmetry

$$\nabla^2\Phi(r) = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi(r)}{dr}\right) = \frac{1}{r}\frac{d^2[r\Phi(r)]}{dr^2}$$

$$\frac{d^2[r\Phi(r)]}{dr^2} = \left\{\frac{mc}{\hbar}\right\}^2[r\Phi(r)]$$

$$\Phi(r) = g\,\frac{e^{-r/R}}{r}$$

Yukawa potential

$$\Phi(r) = g\,\frac{e^{-r/R}}{r}$$

- g is a constant ("coupling strength")
- R is the range of the force

$$R = \frac{\hbar}{mc} = 1.4\,fm$$

Summary

- Very strong, short-range force between nucleons
- Strongly repulsive at very short separations
- Strong force is spin-dependent and 'charge symmetric'
- Interaction potential between nucleons comprises central and non-central (tensor) components
- Yukawa model views the inter-nucleon force in terms of the exchange of pions (π-mesons)
- (Later we'll revisit this idea in *The Standard Model,* quarks and gluons).