Natural Radioactivity

Natural decay

- All very heavy (Z > 83) nuclei are theoretically unstable to α–decay.

$$M_{parent} > M_{daughter} + m_\alpha$$

- Heavy nuclei can change A by α–decay but can change Z by α– or β–decay.
- Only 4 paths (decay chains, sequences) available for decay to final (stable) end product

- *4n ‘Thorium’ series*
- Parent = $^{232}_{90}Th$
- Half-life = 14 Gyr
- End-product = $^{208}_{82}Pb$

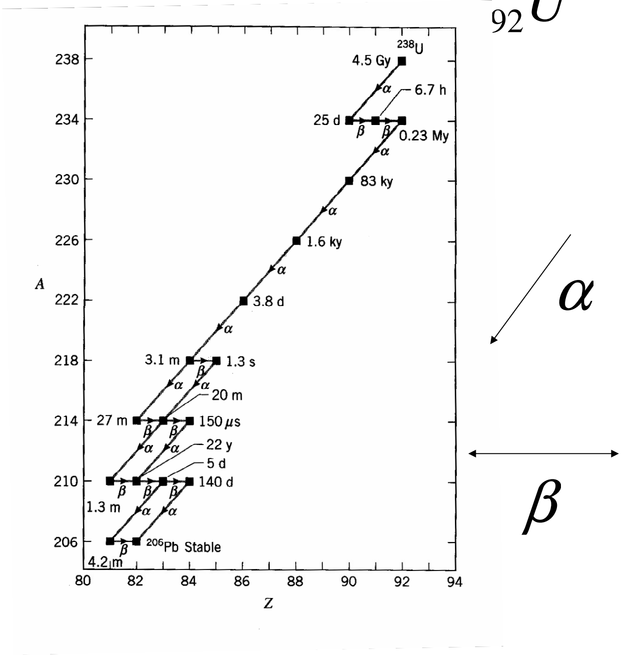
- *4n+1 ‘Neptunium’ series*
- Parent = $^{237}_{93}Np$
- Half-Life = 2.14 Myr (<< Earth)
- End-product = $^{209}_{83}Bi$
- Not found in Nature

- *4n+2 ‘Uranium’ series*
- Parent = $^{238}_{92}U$
- Half-life = 4.47 Gyr
- End-product = $^{206}_{82}Pb$

- *4n+3 ‘Actinium’ series*
- Parent = $^{235}_{92}U$
- Half-Life = 0.7 Gyr
- End-product = $^{207}_{82}Pb$

Decay Chains

$^{238}_{92}U$



$^{206}_{82}Pb$

Rohlf 11.13

Nuclear Reactions

- A particle incident on a nucleus can be ….
- Scattered elastically
- Scattered inelastically (leaves nucleus in an excited energy state which decays by emitting photons or other particles)
- Absorbed (and another particle or particles are emitted)

$$a + X \rightarrow Y + b + Q$$

$$\downarrow$$

$$X(a,b)Y$$

Conservation of energy

$$m_a c^2 + K_a + M_X c^2$$
$$= m_b c^2 + K_b + M_Y c^2 + K_Y$$

$$\therefore Q = (m_i - m_f) c^2$$
$$= (m_a + M_X - m_b - M_Y) c^2$$
$$= K_b + K_Y - K_a$$

assumes initial nucleus X is at rest

- Exothermic (exoergic) (energy released) $Q > 0$
- Endothermic (endoergic). Reaction does not occur unless the incident particle can supply enough energy (KE). $Q < 0$
- Elastic collision $Q = 0$

$$X(a,a)X$$

- Inelastic collision $Q < 0$

$$X(a,a)X^*$$

Threshold energy

- A reaction will not occur if Q < 0, unless the incident particle can supply enough energy (its kinetic energy)

$$K_{th} = -Q\left(\frac{m_a + M_X}{M_X}\right)$$

- N.B. K > /Q/ because the products must have some KE due to conservation of momentum
- Work in Centre of Mass frame (easier because the net momentum is always zero ---- no external forces).

Compound Nucleus

- 1936. Bohr – nuclear reactions take place via the formation and subsequent decay of a compound nucleus.
- The 'Compound Nucleus' is a highly excited composite of the incident particle and the target nucleus (i.e. *a* and *X*).
- Energy is shared by all the nucleons of the Compound Nucleus
- Eventually, one particle gets enough energy to escape

Compound Nucleus

$$\left.\begin{array}{l} p + {}^{15}_{7}N \\ d + {}^{14}_{7}N \\ {}^{3}_{2}He + {}^{13}_{6}C \\ \alpha + {}^{12}_{6}C \\ {}^{6}_{3}Li + {}^{10}_{5}B \end{array}\right\} \quad \boxed{{}^{16}_{8}O^*} \quad \left\{\begin{array}{l} n + {}^{15}_{8}O \\ p + {}^{15}_{7}N \\ d + {}^{14}_{7}N \\ t + {}^{13}_{7}N \\ {}^{3}_{2}He + {}^{13}_{6}C \\ \alpha + {}^{12}_{6}C \\ {}^{6}_{3}Li + {}^{10}_{5}B \\ \gamma + {}^{16}_{8}O \end{array}\right.$$

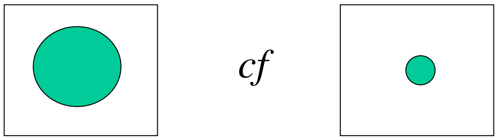
Entrance Channels

Exit Channels

Adapted from Thornton & Rex 13.6

Cross-section

- The probability of a reaction occurring is stated as a "cross section"
- Define an "effective size" for the target nucleus



$$\sigma = \frac{R}{I}$$

reactions per unit time per nucleus

particles incident per unit time per unit area (intensity)

Cross-section

- e.g. bombard $^{13}_{6}C$ with protons
- Some possible reactions are:

$$^{13}_{6}C(p,n)^{13}_{7}N$$
$$^{13}_{6}C(p,\gamma)^{14}_{7}N$$
$$^{13}_{6}C(p,\alpha)^{10}_{5}B$$

- Each possible reaction has its own cross-section called the "partial cross-section"
- 1 barn (b) = 10^{-28} m^2

Cross section

- Scattering probability is proportional to the product of the cross section times the total number of target nuclei
- Normalize by dividing by the target area

$$P = \frac{N_X \sigma}{A} = \frac{ntA\sigma}{A} = nt\sigma$$

n = number of target nuclei per unit volume
t = target thickness

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e.g. Black et al (1968). $^{12}_{6}C(\alpha,n)^{15}_{8}O$

$$E_\alpha = 14.6 MeV$$
$$\sigma = 25\,mb$$
$$1\mu A; \quad 4\,mm^2; \quad t = 1\mu m; \quad \rho = 1.9\,g/cm^3$$

Number of neutrons produced in 1 hour = ?

Number of C atoms per unit volume:

$$n = \frac{\rho N_A}{MW} = 9.53\times10^{28}\ atoms/m^3$$

Scattering probability:

$$P = nt\sigma$$
$$= (9.53\times10^{28})(10^{-6})(25\times10^{-31})$$
$$= 2.4\times10^{-7}$$

α-particle current: $I_\alpha = 10^{-6} A = 10^{-6} C/s$

$$q_\alpha = 2e = 3.2\times10^{-19} C$$

$$N_\alpha = \frac{10^{-6} C/s \times 3600 s}{3.2\times10^{-19} C} = 1.1\times10^{16}$$

$$\begin{aligned} N_n &= N_\alpha P = N_\alpha n t \sigma \\ &= (1.1\times10^{16})(9.53\times10^{28})(10^{-6})(25\times10^{-31}) \\ &= 2.6\times10^{9} \end{aligned}$$

Nuclear Fission

Fission

Binding energy curve

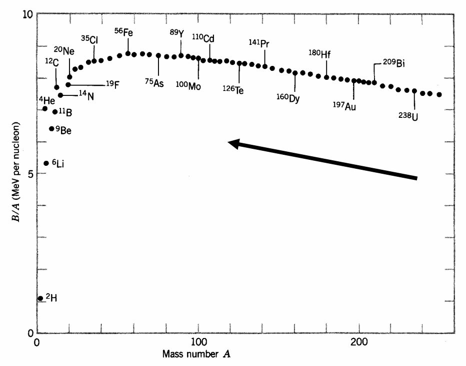


Figure 3.16 The binding energy per nucleon.

$$e.g. \quad 7.6 \rightarrow 8.5\ MeV/nucleon$$

e.g.

$${}^{235}_{92}U + n \rightarrow {}^{92}_{36}Kr + {}^{142}_{56}Ba + 2n + 179.4 MeV$$

1938: Hahn & Strassmann

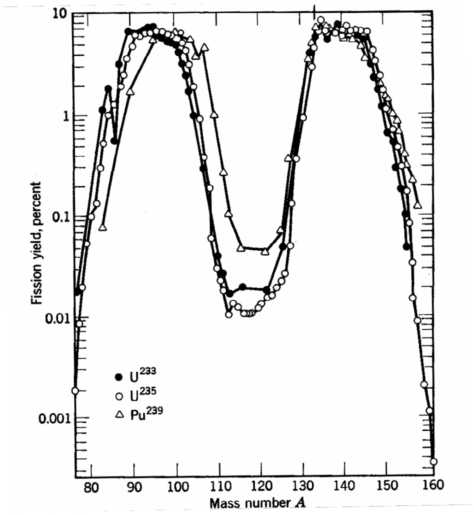
- ${}^{235}_{92}U$ can capture a slow (“thermal”) neutron
- 0.72 % of natural U is U-235
- ‘fast’ neutrons are not captured so the neutron must be slowed “moderated”
- Forms an unstable compound nucleus which splits (“fissions”) into two unequal fragments, with a release of 2 or 3 neutrons (average of 2.4)
- Energy is released

$$M_{after} < M_{before}$$

- Fission occurs because the total Coulombic repulsion of the protons in the nucleus is reduced if the nucleus splits into two fragments
- The preference for ‘magic numbers’ related to uneven fragmentation (still a subject of investigation)

Fission

Mass distribution of fission fragments



16.42 Eisberg & Resnick

Energy release in fission

e.g.

$$^{235}_{92}U + n \rightarrow {}^{92}_{36}Kr + {}^{142}_{56}Ba + 2n + 179.4MeV$$

$$M(^{235}U) = 235.043924\,u$$
$$M(^{92}Kr) = 91.926270\,u$$
$$M(^{142}Ba) = 141.916361\,u$$
$$Q = 235.043924$$
$$-(91.926270 + 141.916361)$$
$$-m_n$$
$$\therefore Q = (1119.006832 - 939.6)MeV$$
$$= 179.4MeV$$

Thermal neutron energy is ~ 1 eV
Energy release is ~ 200 million times the
Incident neutron energy
Chemical combustion ~ 4 eV

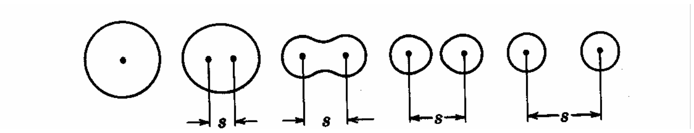
Energy release in fission

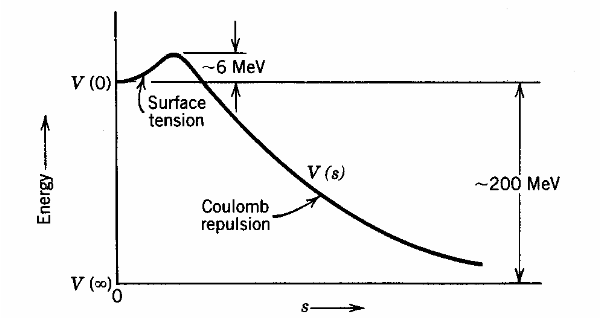
• KE of two fission fragments	• 168 MeV
• Immediate (“prompt”) gamma-rays	• 7 MeV
• Delayed gamma-rays	• 3 – 12 MeV
• Fission neutrons	• 5 MeV
• Gamma rays	• 7 MeV
• Beta particles	• 8 MeV
• Neutrons	• 12 MeV
	• TOTAL ~ 215 MeV

HyperPhysics

Fission – Liquid-Drop Model

Bohr and Wheeler





16.37, 38 Eisberg & Resnick

Energy Content

- Coal (1 kg) → 3×10^7 J
- Oil (1 barrel/0.16 m^3) → 6×10^9 J
- Natural Gas (1 m^3) → 4×10^7 J
- Uranium (1 kg) → 10^{14} J
- Deuterium fusion (1 kg) → 2×10^{14} J

Nuclear Fusion

Fusion

- fuse light nuclei together
- product mass is less than the sum of the constituent masses
- energy released

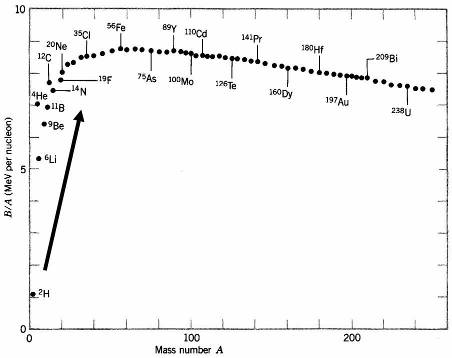


Figure 3.16 The binding energy per nucleon.

Thermonuclear Fusion

- To fuse two protons, one must overcome the Coulomb repulsion (barrier)
- e.g. two protons 5 fm apart

$$U = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$= (9\times10^{9})(1.6\times10^{-19})^2/(5\times10^{-15})$$

$$= 4.6\times10^{-14}\,J \quad (288\,keV)$$

- We have to provide thermal energy of 144 keV to each proton

$$\langle K\rangle = \tfrac{3}{2}k_B T = 2.4\times10^{-14}\,J$$

$$\therefore \quad T = \tfrac{2}{3}(2.4\times10^{-14}\,J)/(1.381\times10^{-23}\,J/K)$$

$$= \quad 1.2\times10^{9}\,K$$

Fusion

- Hydrogen fusion reactions
- “Deuterium Cycle”

$$d + d \rightarrow {}^{3}_{2}He + n + 3.3MeV$$

$$d + d \rightarrow {}^{3}_{1}H\ (=t) + p + 4.0MeV$$

$$d + t \rightarrow {}^{4}_{2}He + n + 17.6MeV$$

$$d + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + p + 18.3MeV$$

The d–t reaction is promising (17.6 MeV) but requires T ~ 40 million K to overcome the Coulomb barrier

Fusion

- Tritium does not occur in nature (10 year half-life)
- Need to ‘breed’ tritium

$${}^{6}_{3}Li + n \rightarrow \alpha + t + 4.8MeV$$

- Deuterium is abundant.
- 1 in 5000 hydrogens in seawater is deuterium

The Sun

- The temperature of the solar core is about 15 million K.

- The nuclear fusion mechanism in The Sun is the proton-proton cycle.

$$p + p \rightarrow d + e^{+} + \nu_e \qquad Q = 0.42\text{ MeV}$$

$$p + d \rightarrow {}^{3}_{2}He + \gamma \qquad Q = 5.49\text{ MeV}$$

$${}^{3}_{2}He + {}^{3}_{2}He \rightarrow p + p + \alpha \qquad Q = 12.86\text{ MeV}$$

- The net result is the ‘burning’ of 4 protons to form an alpha particle + positrons, neutrinos, gammas (Eddington).

Stars

- Most stars are powered by the Carbon (CNO) Cycle, rather than the proton cycle (Stellar core T is > 10^8 K).

$$p + {}^{12}C \rightarrow {}^{13}N + \gamma + 1.94 MeV$$

$${}^{13}N \rightarrow {}^{13}C + e^+ + \nu_e + 1.20 MeV$$

$$p + {}^{13}C \rightarrow {}^{14}N + \gamma + 7.55 MeV$$

$$p + {}^{14}N \rightarrow {}^{15}O + \gamma + 7.29 MeV$$

$${}^{15}O \rightarrow {}^{15}N + e^+ + \nu_e + 1.73 MeV$$

$$p + {}^{15}N \rightarrow {}^{12}C + \alpha + 4.96 MeV$$

- As with the p–p cycle, the net result is the ‘burning’ of 4 protons to form an alpha particle + positrons, neutrinos, gammas (Bethe)

Controlled nuclear fusion

3 main conditions for controlled nuclear fusion

(i) T must be high enough to overcome Coulomb barrier (10^8 K)

(ii) The nuclei must be confined close enough to allow them to fuse (density $n \sim 2–3 \times 10^{20}$ m^{-3})

(iii) Confinement must be sufficiently long ($\tau \sim 1–2$ s)

Lawson criterion (“breakeven”)

$$n \cdot \tau \geq 3 \times 10^{20} \ s / m^3$$

Fusion Product

$$n \cdot \tau \cdot T \geq 6 \times 10^{28} \ s \cdot K / m^3$$