THE UNIVERSITY OF NEW SOUTH WALES
SCHOOL OF PHYSICS

PHYS3050 NUCLEAR PHYSICS
TUTORIAL PROBLEMS

1. Using the r α $A^{1/3}$ observation, estimate the average mass density of a nucleus.

$$\rho = \frac{M}{V} = \frac{A\times u}{\frac{4}{3}\pi r^3} = \frac{A\times 1.66\times 10^{-27}}{\frac{4}{3}\pi (R_o A^{1/3})^3} = \frac{3\times 1.66\times 10^{-27}}{4\pi\times (1.2\times 10^{-15})^3} = 2.3\times 10^{17}\ kg\, m^{-3}$$

It is interesting to compare this value with the highest 'atomic' density found in the Periodic Table which is ~ 22600 kg m^{-3} for the metals Osmium and Iridium, a factor of 10 billion times smaller !

2. [Ashby & Miller 13.3]. Estimate the rest energy of 1 $Å^3$ of nuclear matter.

$$E = mc^2 = \rho V c^2 = 2.3\times 10^{17}\times 10^{-30}\times 9\times 10^{16} = 20.7\ kJ$$

3. [A&M 13.8]. Calculate the distance of closest approach of a 12 MeV deuteron to a Silver nucleus.

$$E = 12\times 10^6\times 1.6\times 10^{-19}\, J = \frac{q_d q_{Ag}}{4\pi\varepsilon_o r} = \frac{9\times 10^9\times 1\times 47\times (1.6\times 10^{-19})^2}{r}\ \therefore r = 5.64\times 10^{-15} m$$

4. [Krane 3.9]. Calculate the binding energy per nucleon of (a) ^{7}Li and (b) ^{56}Fe.

$$B(^7Li) = \left[3m(^1H) + 4m_n - m(^7Li)\right]c^2 = \left[3(1.00782504u) + 4(1.008665u) - 7.016003u\right](931.5\ MeV/u)$$
$$= 39.246 MeV \quad \therefore \quad B/A = 39.246/7 = 5.607 MeV$$
$$B(^{56}Fe) = \left[26m(^1H) + 30m_n - m(^{56}Fe)\right]c^2$$
$$= 492.262 MeV \quad \therefore \quad B/A = 492.262/56 = 8.790 MeV$$

5. [K 3.11]. Calculate the mass defect of ^{238}U.

$$\Delta = \left[m(^{238}U) - 238\right]c^2 = (238.050785 - 238)\times 931.5\ MeV/u$$
$$= 47.306 MeV$$

6. [K 3.13]. Calculate the neutron separation energy of ^{91}Zr.

$$S_n = \left[m(^{90}Zr) - m(^{91}Zr) + m_n\right]c^2 = (89.904703u - 90.905644u + 1.008665u)\times 931.5\ MeV/u$$
$$= 7.195 MeV$$

7. [K 3.13]. Calculate the proton separation energy of ^{197}Au.

$$S_p = \left[m(^{196}Pt) - m(^{197}Au) + m(^1H)\right]c^2 = (195.964926u - 196.966543u + 1.007825u)\times 931.5\ MeV/u$$
$$= 5.783 MeV$$

8. [A&M 14.7] A π^o meson decays into two gamma-rays. If the π^o is at rest, calculate the energy of each gamma-ray.

$$E_\gamma = \tfrac{1}{2} m_{\pi^o} c^2 = \tfrac{1}{2}\times 134.9745\ MeV = 67.4872 MeV$$

9. [K 4.1]. Calculate the minimum photon energy necessary to dissociate the deuteron i.e. $\gamma + d \rightarrow p + n$. Take the deuteron binding energy to be 2.224589 MeV and use a non-relativistic approach.

Conserve energy and linear momentum:

$$p_\gamma = p_p + p_n \quad \& \quad E_\gamma + m_d c^2 = m_p c^2 + K_p + m_n c^2 + K_n$$

$$\rightarrow E_\gamma = [m_p + m_n]c^2 \cdot \left\{1 - \sqrt{\frac{2m_d}{m_p + m_n} - 1}\right\} = 2.226219 MeV$$

10. [A&M 14.11]. On the basis of tabulated masses, which of the isobars, ^{17}N, ^{17}O and ^{17}F, would you expect to be the most stable ? Atomic masses are:

$$M(^{17}N) = 17.008450\,u, \quad M(^{17}O) = 16.999131\,u, \quad M(^{17}F) = 17.002095\,u,$$

^{17}O has the lowest mass and therefore the highest binding energy.

11. [K 5.1] Give the expected shell-model spin and parity assignments for the ground states of (a) ^{7}Li; (b) ^{11}B; (c) ^{15}C; (d) ^{17}F; (e) ^{31}P and (f) ^{141}Pr. [(a) 3/2 and –1; (b) 3/2 and –1; (c) 5/2 and +1; (d) 5/2 and +1; (e) 1/2 and +1; (f) 5/2 and +1]. See left-hand side of figure 5.6, p123.

$$(a) \quad {}^{7}Li, Z = 3, N = 4. \quad p \text{ in } 1p_{3/2} \quad \therefore I = 3/2, \pi = (-1)^l = (-1)^1 = -1 \therefore I^\pi = \tfrac{3}{2}^-$$

$$(b) \quad {}^{11}B, Z = 5, N = 6. \quad p \text{ in } 1p_{3/2} \quad \therefore I = 3/2, \pi = (-1)^l = (-1)^1 = -1 \therefore I^\pi = \tfrac{3}{2}^-$$

$$(c) \quad {}^{15}C, Z = 6, N = 9. \quad n \text{ in } 1d_{5/2} \quad \therefore I = 5/2, \pi = (-1)^l = (-1)^2 = +1 \therefore I^\pi = \tfrac{5}{2}^+$$

$$(d) \quad {}^{17}F, Z = 9, N = 8. \quad p \text{ in } 1d_{5/2} \quad \therefore I = 5/2, \pi = (-1)^l = (-1)^2 = +1 \therefore I^\pi = \tfrac{5}{2}^+$$

$$(e) \quad {}^{31}P, Z = 15, N = 16. \quad p \text{ in } 2s_{1/2} \quad \therefore I = 1/2, \pi = (-1)^l = (-1)^0 = +1 \therefore I^\pi = \tfrac{1}{2}^+$$

$$(f) \quad {}^{141}\text{Pr}, Z = 59, N = 82. \quad p \text{ in } 2d_{5/2} \quad \therefore I = 5/2, \pi = (-1)^l = (-1)^2 = +1 \therefore I^\pi = \tfrac{5}{2}^+$$

12. [K5.7] Calculate the shell-model quadrupole moment of ^{209}Bi (I = 9/2$^-$). (Experimental value = –0.37 b).

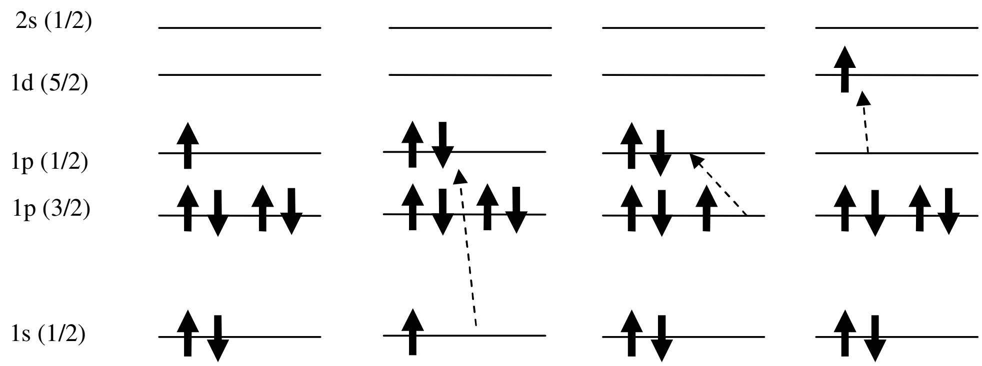
$$\langle Q \rangle = \langle Q_{sp} \rangle = -\frac{2j-1}{2(j+1)}\langle r^2 \rangle = -\frac{2j-1}{2(j+1)} \cdot \frac{3}{5}R^2 = -\frac{2j-1}{2(j+1)} \cdot \frac{3}{5}R_0^2 A^{2/3}$$

$$= -\left(\frac{2 \cdot \frac{9}{2} - 1}{2 \cdot \frac{11}{2}}\right) \times \frac{3}{5} \times 1.2^2 \times 209^{2/3} = -22.13\ fm^2 = -0.22b$$

13. [K5.2] The low-lying energy levels of ^{13}C are: Ground state (1/2–); 3.09 MeV (1/2+); 3.68 MeV (3/2–) and 3.85 MeV (5/2+). Interpret these states according to the shell-model.

$$^{13}C, Z = 6, N = 7.$$

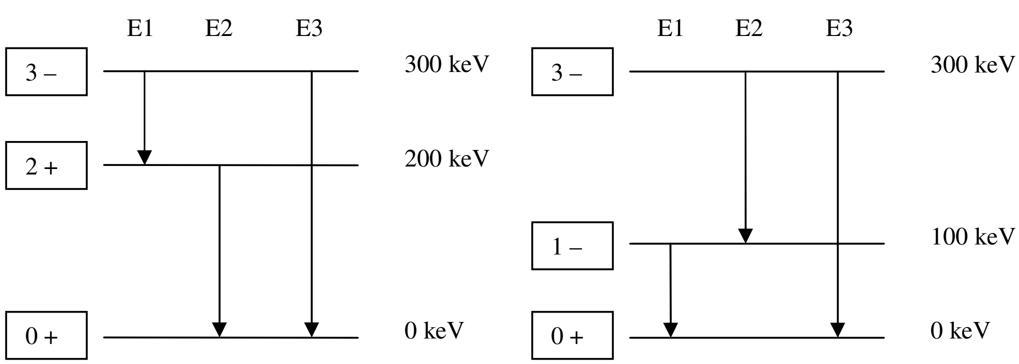
Protons are paired; the energy states of ^{13}C are determined by the unpaired (7th) neutron.



Therefore, the energy states are (L to R): $\frac{1}{2}^-, \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+$

14. [K10.7]. A certain decay process leads to final states in an even-Z, even-N nucleus and gives only three γ rays of energies 100, 200 and 300 keV, whose multipolarities are E1, E2 and E3, respectively. Construct two possible level schemes for this nucleus and label the states with their most likely spin-parity assignments.

The parity of the En γ-ray is $(-1)^n$ so E1 and E3 involve a change in parity between the two nuclear levels involved. E2 involves no such change. Furthermore, the ground state of the even-even nucleus is 0^+. The 'n' in En gives the change in nuclear spin.



15. [Serway et al 15.17]. By considering the quark makeup of the various particles, deduce the identity of the unknown particle in the reaction

$$\Sigma^0 + p \rightarrow \Sigma^+ + \gamma + ? \qquad \begin{array}{l} \Sigma^0 = uds;\ p = uud;\ \Sigma^+ = uus \\ u: 3 \rightarrow 2 + ?\ \therefore u \\ d: 2 \rightarrow 0\ \therefore dd \\ s: 1 \rightarrow 1 \end{array}$$

Mystery particle has quark makeup *udd* i.e. neutron.

16. [K9.9]. The complete processes are:

$$\bar{\nu} + {}^3He \rightarrow {}^3H + e^+ \qquad {}^6He \rightarrow {}^6Li + e^- + \bar{\nu}$$

$$e^- + {}^8B \rightarrow {}^8Be + \nu \qquad \nu + {}^{12}C \rightarrow {}^{12}N + e^-$$

$${}^{40}K \rightarrow \nu + e^+ + {}^{40}Ar \qquad {}^{40}K \rightarrow \bar{\nu} + e^- + {}^{40}Ca$$

17. [K9.4]. The maximum kinetic energy of the positron spectrum emitted in the decay ${}^{11}C \rightarrow {}^{11}B$ is 1.983±0.003 MeV. Use this information and the known mass of ${}^{11}B$ to calculate the mass of ${}^{11}C$.

$${}^{11}_{6}C \rightarrow {}^{11}_{5}B + \beta^+ + \nu_e$$

$$Q = K_\beta^{\max} = 1.983(3)\ MeV = \left[m({}^{11}C) - m({}^{11}B) - 2m_e\right]c^2$$

$$\therefore m({}^{11}C) = \left(\frac{Q}{c^2}\right) + m({}^{11}B) + 2m_e = \left\{\frac{1.983(3)\,MeV}{931.502\,MeV/u}\right\} + 11.009305u + 2(5.485803\times10^{-4}) = 11.012531(3)\,u$$

18. [Serway 45.59]. A by-product of some fission reactors is ^{239}Pu which is an α-emitter with a half-life of 24,120 years. Consider 1 kg of ^{239}Pu at t=0. (a) What is the number of ^{239}Pu nuclei at t=0 ? (b) What is the initial activity ? (c) For how long would you need to store the Plutonium until it had decayed to a safe activity level of 0.1 Bq ?

$$(a) \quad 1\text{ nucleus} = 239\times1.66\times10^{-27} = 3.97\times10^{-25}\ kg\ \therefore N_0 = 1/(3.97\times10^{-25}) = 2.519\times10^{24}$$

$$(b) \quad R_0 = \lambda N_0 = \frac{\ln 2}{t_{1/2}} N_0 = 2.305\times10^{12}\,Bq$$

$$(c) \quad 0.1 = 2.305\times10^{12} e^{-\lambda t}\ \therefore t = 1.07\times10^6\ yrs$$

19. [A&M 15.14]. Calculate the Q value for the β^- decay of ^{108}Ag.

$${}^{108}_{47}Ag \rightarrow {}^{108}_{48}Cd + \beta^- + \bar{\nu}_e$$

$$Q = (M_P - M_D)c^2 = (107.905952 - 107.904176)u \times 931.5MeV/u = 1.65MeV$$

20. [A&M 15.19]. On the basis of Q values, determine if the ^{98}Tc nucleus can decay by (a) β^- decay, (b) β^+ decay, (c) electron capture.

(a) ${}^{98}_{43}Tc \rightarrow {}^{98}_{44}Ru + \beta^- + \bar{\nu}_e$

$$Q = (M_P - M_D)c^2 = (97.907215 - 97.905287)u \times 931.5MeV/u = 1.796MeV \quad i.e. \quad > 0\ \therefore Yes$$

(b) $^{98}_{43}Tc \rightarrow ^{98}_{42}Mo + \beta^+ + \nu_e$

$$Q = (M_P - M_D - 2m_e)c^2 = (97.907215 - 97.905407 - 2\times 5.485803\times 10^{-4})u \times 931.5 MeV/u = 0.662 MeV$$
$i.e. \quad > 0 \therefore Yes$

(c) $e^- + ^{98}_{43}Tc \rightarrow ^{98}_{42}Mo + \nu_e$

$$Q = (M_P - M_D)c^2 = (97.907215 - 97.905407)u \times 931.5 MeV/u = 1.684 MeV \quad i.e. \quad > 0 \therefore Yes$$

21. [S 46.31]. A 2 MeV neutron is emitted in a fission reactor. If it loses half of its kinetic energy in each collision with a moderator atom, how many collisions must it undergo to achieve thermal energy (0.039 eV) ?

$$2^n = \frac{2\times 10^6}{0.039} = 5.13\times 10^7 \quad \therefore n = 25.6 \rightarrow 26$$

22. [S 46.37]. Assume a deuteron and a triton are at rest when they fuse according to $d + t \rightarrow \alpha + n$. This reaction has a Q value of 17.6 MeV. Determine the kinetic energies acquired by the α and the *n*.

$$p_\alpha = p_n \quad \& \quad \frac{p_\alpha^2}{2m_\alpha} + \frac{p_n^2}{2m_n} = 17.6 MeV = K_\alpha + K_n \rightarrow K_\alpha = 3.5 MeV \;\&\; K_n = 14.1 MeV$$

23. [Rohlf 11.15]. If the activity of a substance drops by a factor of 32 in 5 seconds, what is its half-life ? $32 = 2^5 \therefore 5s = 5\times t_{1/2} \therefore t_{1/2} = 1s$

24. [R 11.26]. Show that the decay $n \rightarrow p + e^-$ cannot conserve angular momentum.

All particles have spin 1/2. Two spin-1/2s cannot be combined to yield s=1/2.

25. [R 17.16]. Which of the following strong interactions are allowed ? If a process is forbidden, state the reason.

Reaction	Electric Charge	Baryon #	Strangeness
$\bar{p} + p \rightarrow \pi^0 + n$	Yes	**No**	Yes
$\pi^- + p \rightarrow K^0 + n$	Yes	Yes	**No**
$p + p \rightarrow \pi^+ + n + n$	**No**	Yes	Yes
$\bar{p} + p \rightarrow \pi^0 + \pi^+ + \pi^-$	Yes	Yes	Yes
$K^- + p \rightarrow \pi^0 + \Lambda^0$	Yes	Yes	Yes

26. [K 18.6]. Analyse the following decays or reactions for possible violations of the basic conservation laws. In each case, state which conservation laws, if any, are violated and through which interaction the process will most likely proceed (if at all):

$(a)\quad \pi^+ + p \to p + p + \bar{n}$

$(b)\quad \Sigma^+ \to n + e^+ + \nu_e$

$(c)\quad K^+ \to \pi^+ + e^+ + e^-$

$(d)\quad \pi^- + p \to \Lambda^0 + \Sigma^0$

$(e)\quad K^+ \to \pi^+ + \pi^+ + \pi^0 + \pi^-$

$(f)\quad K^+ \to \pi^+ + e^+ + \mu^-$

$(g)\quad \Lambda^0 + p \to \Sigma^+ + n$

$(h)\quad \Lambda^0 \to p + K^-$

(a) conserves B, L_e, L_μ, S, Q, T & T_3; proceeds via the strong interaction

(b) violates S; proceeds via the weak interaction

(c) violates S; proceeds via the weak interaction

(d) violates B, S, T & T_3; forbidden due to B-violation

(e) violates conservation of energy so forbidden

(f) violates L_e & L_μ and is forbidden

(g) conserves B, L_e, L_μ, S, Q, T & T_3; proceeds via the strong interaction

(h) violates conservation of energy so forbidden

27. [K 18.7]. Analyse the following decays according to their quark content:

$(a)\quad \Omega^- \to \Lambda^0 + K^-$

$(b)\quad K^+ \to \pi^+ + \pi^0$

$(c)\quad \Xi^- \to \Lambda^0 + \pi^-$

$(d)\quad \Lambda_c^+ \to p + \overline{K}^0$

$(a)\quad \Omega^- = sss;\ \Lambda^0 = uds;\ K^- = \bar{u}s \qquad sss \to uds + \bar{u}s \quad \therefore s \to d$ & a $\bar{u}u$ pair created

$s \to u + W^- \quad \& \quad W^- \to \bar{u} + d$

$(b)\quad K^- = \bar{u}s; \pi^+ = u\bar{d}; \pi^0 = u\bar{u} + d\bar{d}; \quad u\bar{s} \to u\bar{d} + (u\bar{u} + d\bar{d}); \quad \bar{s} \to \bar{u} + W^+ \quad \& \quad W^+ \to u + \bar{d}$

$(c)\quad \Xi^- = dss;\ \Lambda^0 = uds;\ \pi^- = \bar{u}d \qquad dss \to uds + \bar{u}d \quad \therefore s \to u + W^- \quad \& \quad W^- \to \bar{u} + d$

$(d)\quad \Lambda_c^+ = udc;\ p = uud;\ \overline{K}^0 = \bar{d}s \qquad udc \to uud + \bar{d}s \quad \therefore c \to s + W^+ \quad \& \quad W^+ \to u + \bar{d}$

28. [K 18.8]. Analyse the following reactions according to their quark content:

$(a)\quad K^- + p \to \Omega^- + K^+ + K^0$ $\qquad (c)\quad K^- + p \to \Xi^- + K^+$

$(b)\quad p + p \to p + \pi^+ + \Lambda^0 + K^0$ $\qquad (d)\quad \pi^- + n \to \Delta^- + \pi^0$

$(a)\quad K^- + p \to \Omega^- + K^+ + K^0$

$\bar{u}s + uud \to sss + u\bar{s} + d\bar{s} \qquad \therefore \quad u\bar{u} \to s\bar{s} + s\bar{s}$

$(b)\quad p + p \to p + \pi^+ + \Lambda^0 + K^0$

$uud + uud \to uud + u\bar{d} + uds + d\bar{s} \qquad \therefore$ create $d\bar{d} + s\bar{s}$

$(c)\quad K^- + p \to \Xi^- + K^+$

$\bar{u}s + uud \to dss + u\bar{s} \qquad \therefore \quad u\bar{u} \to s\bar{s}$

$(d)\quad \pi^- + n \to \Delta^- + \pi^0$

$\bar{u}d + udd \to ddd + (u\bar{u} + d\bar{d}) \qquad \therefore$ create $d\bar{d}$

29. [K 17.4]. Find which of the following reactions are forbidden by one or more conservation laws. Give all violated laws in each case.

$(a)\quad K^+ + n \to \Sigma^+ + \pi^0$ $\qquad (d)\quad \pi^- + p \to \Sigma^+ + K^-$

$(b)\quad \pi^- + n \to K^+ + \Lambda^0$ $\qquad (e)\quad \pi^- + p \to \Xi^- + K^+ + \bar{K}^0$

$(c)\quad K^- + p \to n + \Lambda^0$ $\qquad (f)\quad d + d \to \alpha + \pi^0$

(a) S; (b) Q & T_3; (c) T & T_3; (d) S & T_3; (e) S & T_3 and (f) T