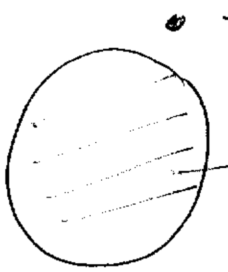
# PART V

1) Shell model magnetic moments. Schmidt lines.

2) Isotopes, stability of nuclei; β-decay, α-decay, fission.

3) Neutron star.



• – external (odd) nucleon.

closed shells (paired nucleons)

$$\vec{\mu} = \mu_N (g_s \vec{s} + g_l \vec{l})$$

$s$ – spin of the external nucleon

$l$ – orbital angular momentum of the external nucleon

$$g_l = \begin{cases} 1 & \text{for } p \\ 0 & \text{for } n \end{cases}$$

see page

$$g_s = \begin{cases} 5.6 & \text{for } p \\ -3.8 & \text{for } n \end{cases}$$

$\mu_N = \frac{|e|\hbar}{2m_p c}$ – nuclear magneton

Because of the spin-orbit interaction only the total angular momentum is good quantum number: $\vec{j} = \vec{l} + \vec{s}$

$$|\psi\rangle = |j, j_z, \ell, s\rangle$$

$$\langle\psi|\vec{\mu}|\psi\rangle = \mu_N\left[g_s\langle\vec{s}\rangle + g_\ell\langle\vec{\ell}\rangle\right] = \mu_N A\vec{J}$$

$g_s\langle\vec{s}\rangle + g_\ell\langle\vec{\ell}\rangle = A\vec{J}$, multiply this eq by $\vec{j}$

$$\Rightarrow g_s\langle\vec{s}\vec{j}\rangle + g_\ell\langle\vec{\ell}\vec{j}\rangle = A\vec{j}^2 \Rightarrow Aj(j+1)$$

$$\begin{cases} \langle\vec{s}\vec{j}\rangle = \langle\vec{s}(\vec{\ell}+\vec{s})\rangle = \langle\vec{s}\vec{\ell}\rangle + \langle s^2\rangle = \langle\vec{s}\cdot\vec{\ell}\rangle + 3/4 \\ \langle\vec{\ell}\vec{j}\rangle = \langle\vec{\ell}(\vec{\ell}+\vec{s})\rangle = \langle\vec{\ell}^2\rangle + \langle\vec{s}\cdot\vec{\ell}\rangle = \ell(\ell+1) + \langle\vec{s}\cdot\vec{\ell}\rangle \end{cases}$$

$$Aj(j+1) = g_s\left[\frac{3}{4} + \langle\vec{s}\cdot\vec{\ell}\rangle\right] + g_\ell\left[\ell(\ell+1) + \langle\vec{s}\cdot\vec{\ell}\rangle\right]$$

Definition of the magnetic moment of the nucleus: $\mu = \langle\vec{\mu}\rangle_{\text{at max } j_z} = \mu_N A j_z =$

$= \mu_N A j$. $\boxed{\text{because } \max j_z = j}$

Hence

$$\boxed{\mu = \mu_N A j = \mu_N \frac{1}{(j+1)}\left[\frac{3}{4}g_s + \ell(\ell+1)g_\ell + (g_s + g_\ell)\langle\vec{\ell}\cdot\vec{s}\rangle\right]}$$

$$\vec{j} = \vec{\ell} + \vec{s} \Rightarrow \vec{j}^2 = \vec{\ell}^2 + 2\vec{\ell}\cdot\vec{s} + \vec{s}^2 \Rightarrow$$

$$\Rightarrow \langle \vec{j}^2 \rangle = \langle \vec{\ell}^2 \rangle + 2\langle \vec{\ell}\cdot\vec{s} \rangle + \langle \vec{s}^2 \rangle$$

or $j(j+1) = \ell(\ell+1) + 2\langle \vec{\ell}\cdot\vec{s} \rangle + 3/4 \Longrightarrow$

$$\Longrightarrow \boxed{\langle \vec{\ell}\cdot\vec{s} \rangle = \frac{1}{2}\left[j(j+1) - \ell(\ell+1) - 3/4\right]}$$

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$\vec{J} = \vec{\ell} + \vec{s}$ , $s = 1/2$ , hence $j = \ell \pm 1/2$

1) $j = \ell + 1/2$

$$\langle \vec{\ell}\cdot\vec{s} \rangle = \frac{1}{2}\left[(\ell+1/2)(\ell+3/2) - \ell(\ell+1) - \frac{3}{4}\right] = \frac{\ell}{2}$$

$$\mu = \mu_N \frac{1}{\ell + 3/2}\left[\frac{3}{4}g_s + \ell(\ell+1)g_\ell + (g_s + g_\ell)\frac{\ell}{2}\right] = \mu_N\left[\frac{1}{2}g_s + \ell g_\ell\right]$$

$$\boxed{\mu = \mu_N\left[\frac{1}{2}g_s + \left(j - \frac{1}{2}\right)g_\ell\right]}$$

2) $j = \ell - 1/2$

$$\langle \vec{\ell}\cdot\vec{s}\rangle = \frac{1}{2}\left[\left(\ell-\frac{1}{2}\right)\left(\ell+\frac{1}{2}\right) - \ell(\ell+1) - \frac{3}{4}\right] = -\frac{\ell+1}{2}$$

$$\mu = \mu_N \frac{1}{\ell+\frac{1}{2}}\left[\frac{3}{4} g_s + \ell(\ell+1) g_\ell - (g_s + g_\ell)\frac{\ell+1}{2}\right] =$$

$$= \mu_N \frac{1}{\ell+\frac{1}{2}}\left[-\frac{1}{2} g_s \left(\ell-\frac{1}{2}\right) + g_\ell \left(\ell-\frac{1}{2}\right)(\ell+1)\right] =$$

$$= \boxed{\mu_N\left[-\frac{1}{2} g_s \frac{j}{j+1} + g_\ell \frac{j(j+3/2)}{j+1}\right]}$$

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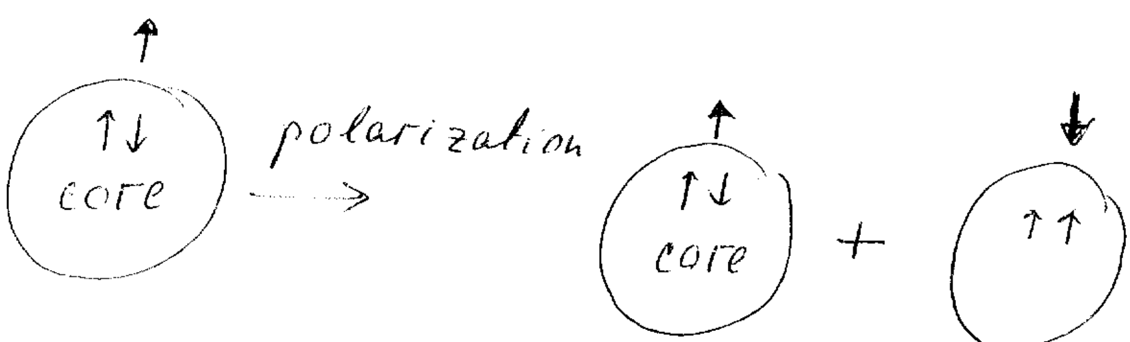
Because of polarization of the core $g_s$ is changed: $g_s \to 0.6 g_s$




polarization is due to the strong interaction

Schmidt lines for neutrons



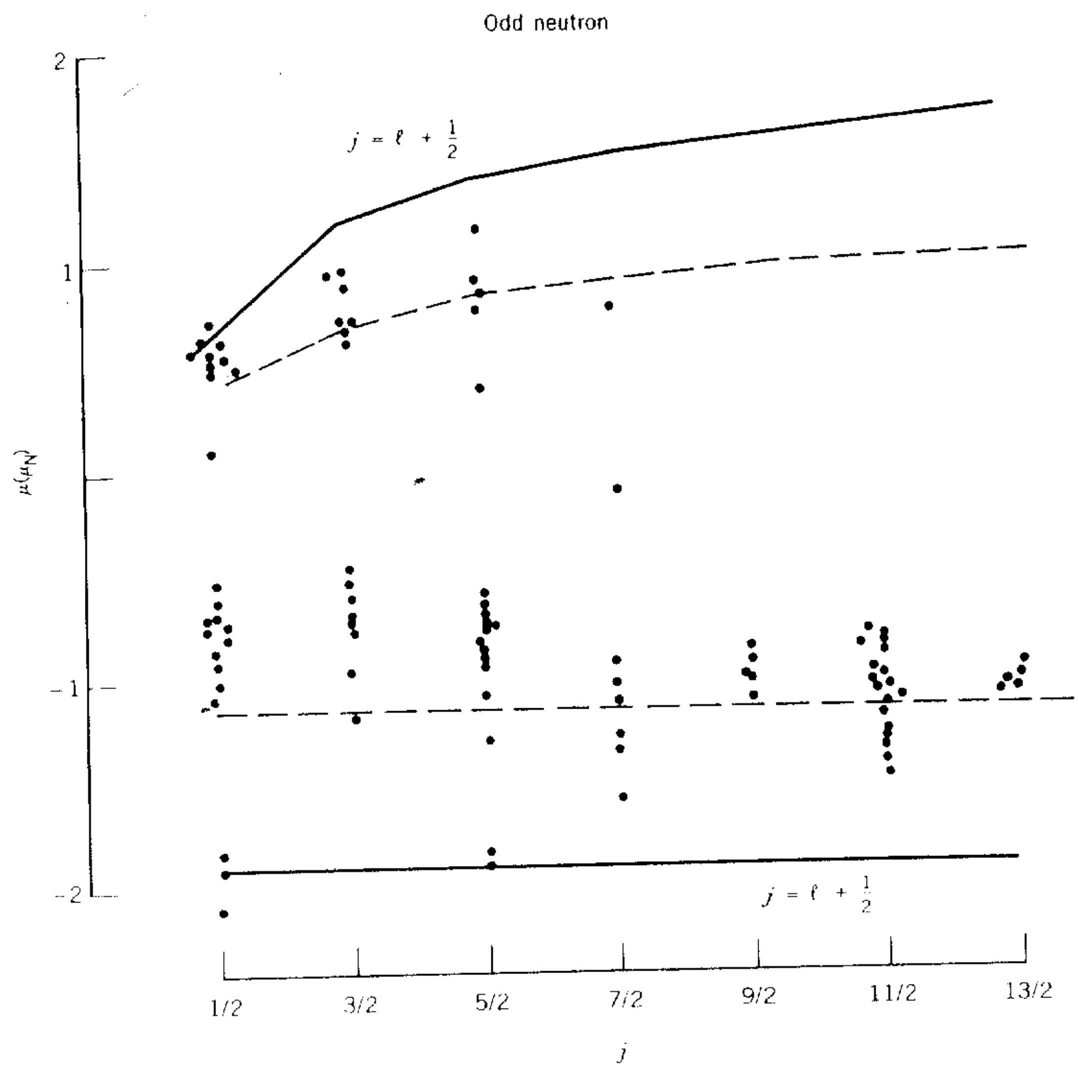



**Figure 5.9** Experimental values for the magnetic moments of odd-neutron and odd-proton shell-model nuclei. The Schmidt lines are shown as solid for $g_s = g_s$(free) and dashed for $g_s = 0.6g_s$(free).

arbitrarily) reducing the $g_s$ factor; for example, the lines for $g_s = 0.6g_s$(free) are shown in Figure 5.9. The overall agreement with experiment is better, but the scatter of the points suggests that the model is oversimplifying the calculation of magnetic moments. Nevertheless, the success in indicating the general trend of the observed magnetic moments suggests that the shell model gives us at least an approximate understanding of the structure of these nuclei.

## Electric Quadrupole Moments

The calculation of electric quadrupole moments in the shell model is done by evaluating the electric quadrupole operator, $3z^2 - r^2$, in a state in which the total angular momentum of the odd particle has its maximum projection along the $z$ axis (that is, $m_j = +j$). Let's assume for now that the odd particle is a proton. If its angular momentum is aligned (as closely as quantum mechanics allows) with the $z$ axis, then it must be orbiting mostly in the $xy$ plane. As we indicated in the discussion following Equation 3.36, this would give a negative quadrupole

Schmidt lines for protons

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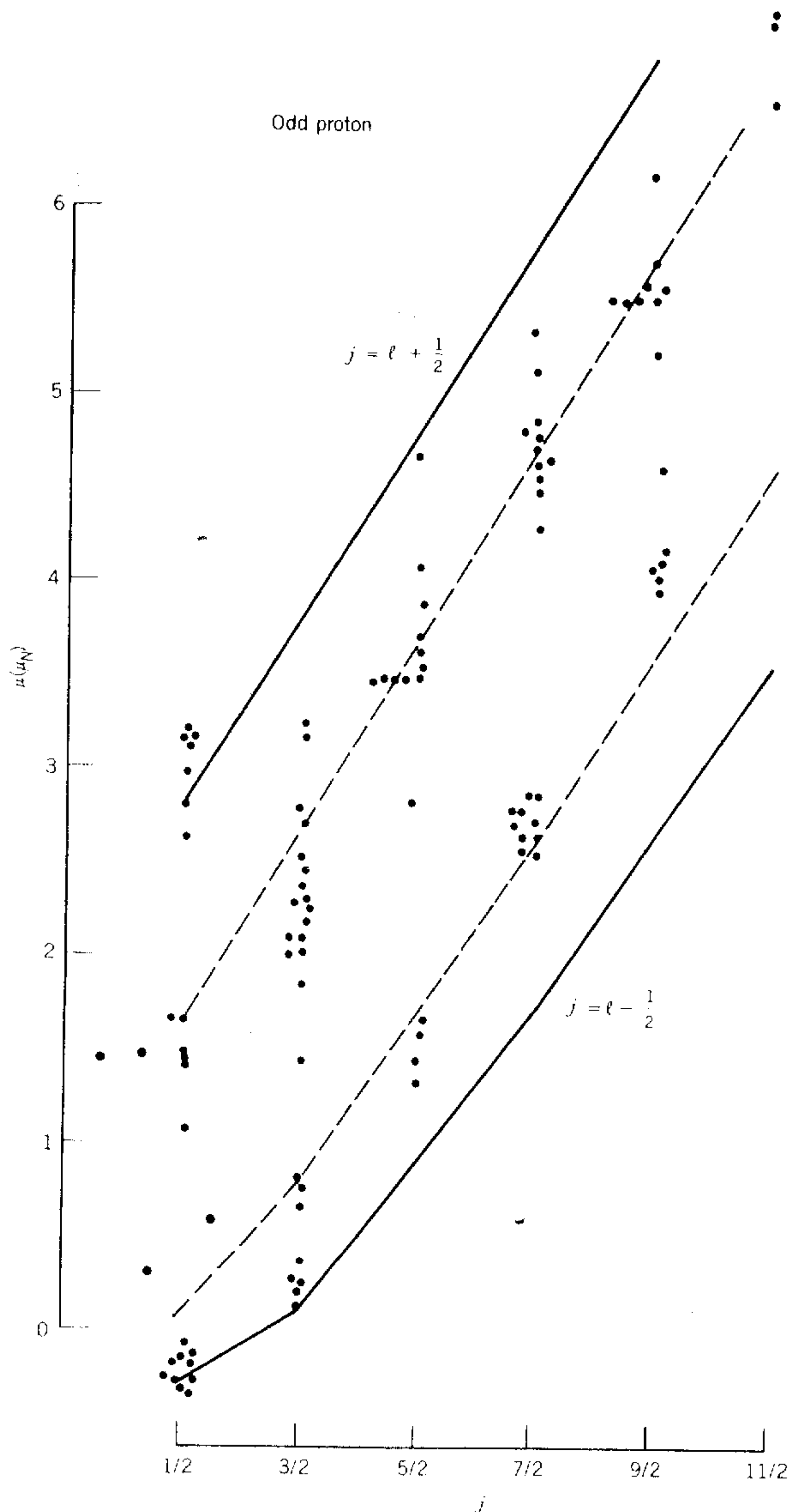
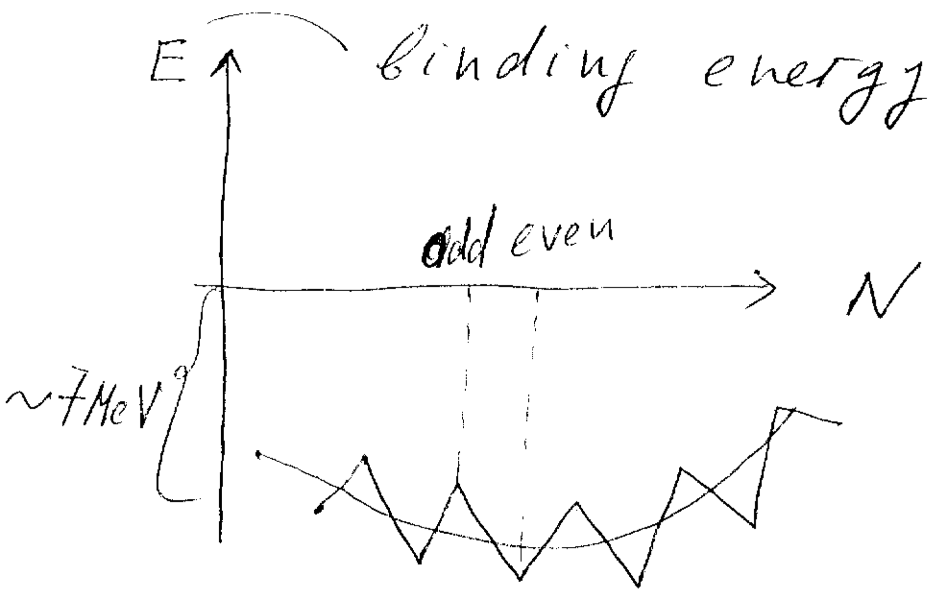



**Figure 5.9** Continued.

# Isotopes

Nuclei with given Z, but different number of neutrons N.

Atomic and chemical properties are defined by Z.




at even N the binding is stronger because of pairing.

Stable isotopes are near minimum of the curve. They correspond to maximum binding.

Without account of the Coulomb repulsion $N=Z$ in stable isotopes because of the isotopic symmetry of strong interaction.

In heavy nuclei, due to the Coulomb repulsion, $N \approx 1.5Z$ in stable isotopes.

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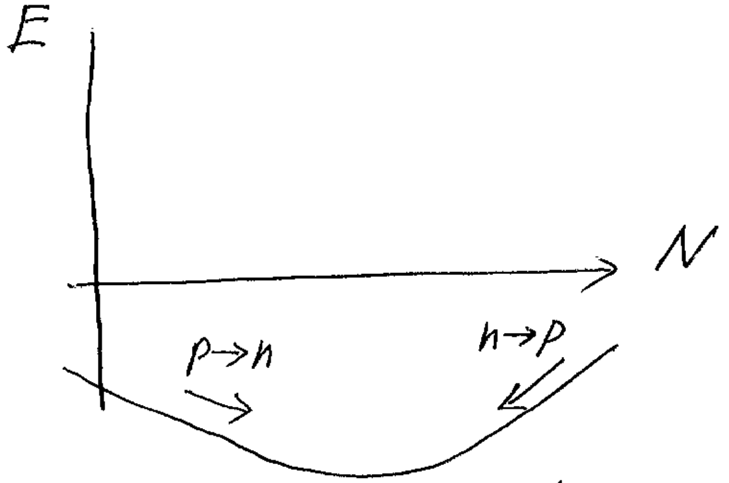
## $\beta$-decay of unstable isotopes

Let us neglect the even-odd effect (i. e. let us smooth the binding curve)




$\beta$-decay due to the weak interaction

$$n \to p + e^- + \tilde{\nu}_e \Longleftrightarrow (N,Z) \to (N-1, Z+1)$$

or $$p \to n + e^+ + \nu_e \Longleftrightarrow (N, Z) \to (N+1, Z-1)$$

$\beta$-decay is possible if the binding energy gain is bigger than the electron rest energy, $\Delta E > m_e c^2 \approx 0.5$ MeV

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Other limitations for stability of nuclei: $\alpha$-decay, fission

$\alpha$-decay – emission of $\alpha$-particle

$\alpha = {}^4He = ppnn$

example: $\begin{cases} {}^{228}Th \rightarrow {}^{224}Ra + \alpha \\ t_{1/2} = 1.91 \text{ year} \end{cases}$

example: $\begin{cases} {}^{256}Cf \rightarrow \text{fission} \\ t_{1/2} = 12.3 \text{ min} \end{cases}$

example: ${}^{238}U$ → $\alpha$ decay, $t_{1/2}(\alpha) = 4.46 \; 10^9$ years
→ fission, $t_{1/2}(f) \approx 10^{16}$ years

$Z = 83$; Bi has the only stable isotope $^{209}Bi$.

There are no stable nuclei at $Z > 83$.

However sometimes lifetime is longer than lifetime of the Universe, see example at previouse page.

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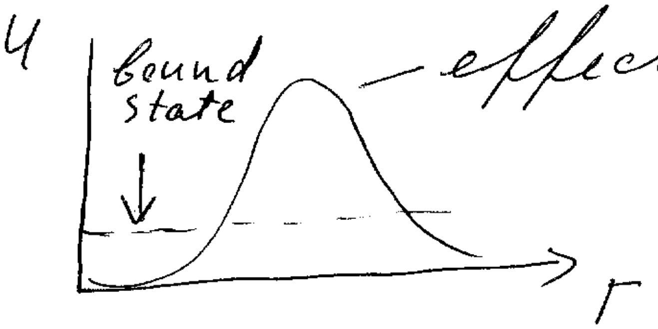
The spread in lifetimes is so large because of the subbarier nature of the decays.




decay due to the subbarier tunneling

$$\text{Probability} \sim \left| e^{-\frac{1}{\hbar}\int p\,dx} \right|^2 = \left| e^{-\int \sqrt{2m(U-E)}\,dx} \right|^2$$

# Neutron star.

Consider a star that collaps gravitationally down to nuclear density. Before collaps the number of neutrons ~ number of protons = = number of electrons.

After the collaps protons capture electrons in the reaction

$p + e^- \rightarrow n + \nu_e$ (similar to $\beta$-decay)

with escape of neutrino.

So after the collapse $\boxed{Z \ll N}$

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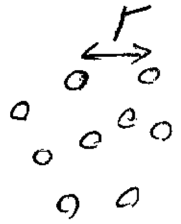
To prove this assume opposite, so assume that $Z \sim N$.




r is average separation between particles.

At nuclear density $r \sim 1\,fm$.
This is separation between nucleons and separation between electrons is the same. Hence typical electron momentum due to the Fermi statistics $p \sim 1/r \sim 200\,MeV$.
Hence energy of the electron
$\varepsilon \sim p \sim 200\,MeV$ (ultrarelativstic)

$$\varepsilon \gg m_n - m_p = 1.3\,MeV$$

Hence the system con reduce energy by capture of electrons in the reaction $p + e^- \to n + \nu$.

Therefore $Z \ll N$.