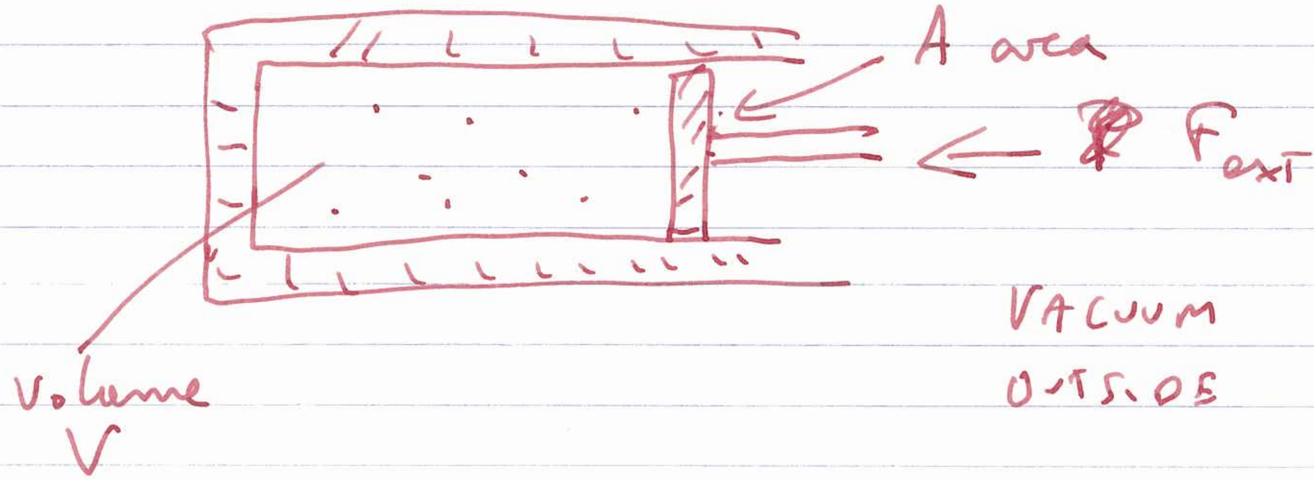
LECTURE 4 ① 7/8/09

KINETIC THEORY of GASES

Aim: ① To link microscopic physics to macroscopic

② To get the eqns of state for a model system: Ideal, (single cpt) monatomic gas

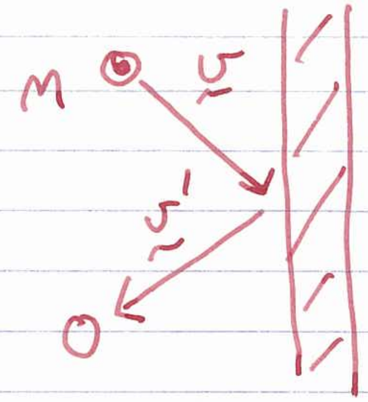
Pressure due to a gas



At mechanical EQM

$$P_{gas} = -P_{ext}$$
$$= -F_{ext}/A$$

What ~~causes~~ is the source of the gas pressure?



particles bounce off the wall

If collision is elastic

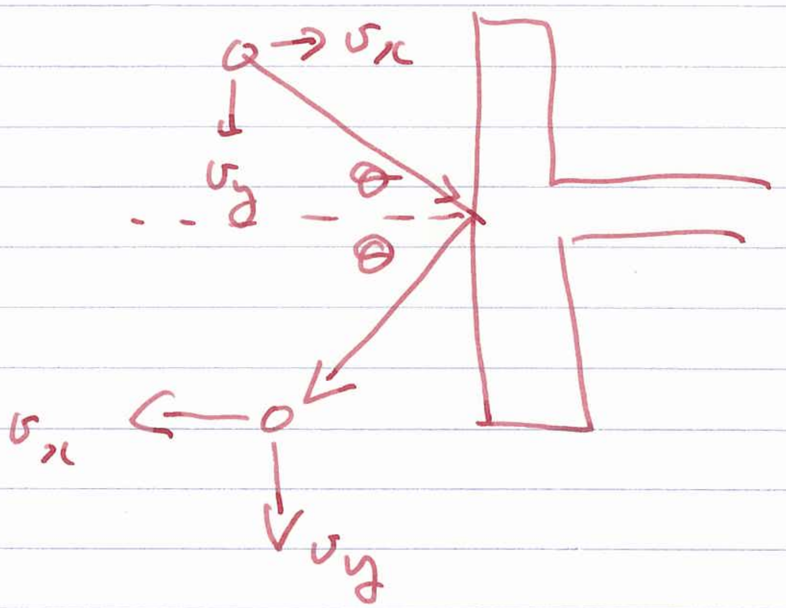
($\equiv$ ΔKE particle $= 0$)

Then

$$\underset{\sim}{F} = \frac{d\underset{\sim}{p}}{dt}$$

where $\underset{\sim}{p}$ = momentum

For one particle:



So ~~total~~ ~~initial~~ velocity of before collision

$$\underset{\sim}{v} = (v_x, v_y, v_z)$$

velocity ~~of the~~ after collision

$$\underset{\sim}{v}' = (-v_x, v_y, v_z)$$

$\therefore$ Change in momentum for a single particle:

$$\Delta \underset{\sim}{p} = \underset{\sim}{p}' - \underset{\sim}{p}$$ of the particle

$$= m(-v_x, v_y, v_z) - m(+v_x, v_y, v_z)$$

$$\boxed{\Delta \underset{\sim}{p} = -2 m v_x \underset{\sim}{i}}$$ of the particle

for a single particle

$$\therefore \boxed{\Delta \underset{\sim}{p}_{wall} = +2 m v_x \underset{\sim}{i}}$$ - per particle

For the gas:

$$\underset{\sim}{F} = \frac{dp}{dt}$$

$$= \frac{dp}{dn_c} \cdot \frac{dn_c}{dt}$$

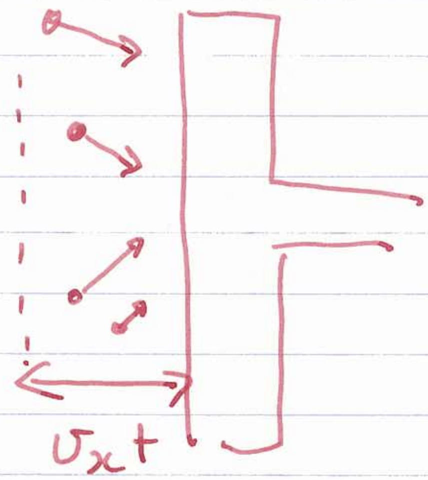
change of momentum per particle colliding with wall

number of particles colliding with wall per unit time

Need

$$\frac{dn_c}{dt}:$$

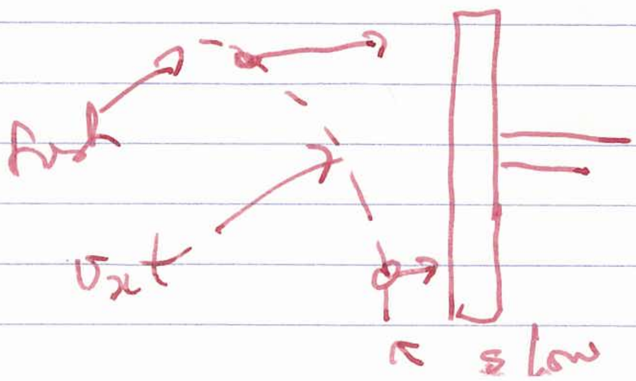
WATCH cylinder wall for time t:



particles moving right within distance $v_x t$ will reach piston & collide within t.

Trouble is: $v_x \neq$ const

– distribution of velocities



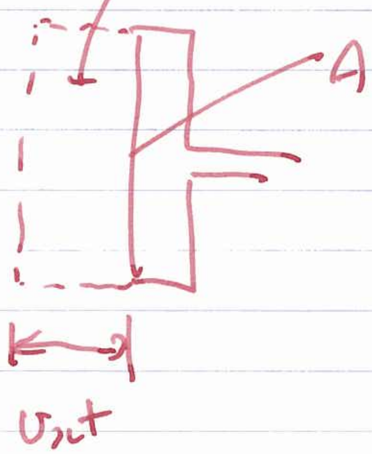
For each particle: if:

1. moving right
& 2. within $v_x t$ of wall

it will collide

$\therefore$ PARTICLES that will collide with wall are in volume

$v_x t A$



particles in this volume:

$$n' = \frac{N}{V} \cdot v_x t A$$

but only those moving right will collide with wall

$$\therefore \quad n_c = \frac{1}{2} \frac{N}{V} \cdot v_x t A$$

$\frac{1}{2}$ left

& $\frac{1}{2}$ right

$$\therefore \quad \boxed{\frac{dn_c}{dt} = \frac{1}{2} \frac{N}{V} \cdot v_x A}$$

$\therefore$ Force Exerted on piston by gas:

$$\underset{\sim}{F} = \frac{dp}{dt}$$

$$= \frac{dp}{dn_c} \cdot \frac{dn_c}{dt}$$

$$= +2m v_x \underset{\sim}{i} \cdot \frac{1}{2} \frac{N}{V} \cdot v_x A$$

$$= + m \frac{N}{V} v_x^2 A$$

$$\therefore \quad \boxed{P = m \frac{N}{V} v_x^2} \qquad \text{since } P = \frac{F}{A}$$

Trouble with derivation:

Assumed all particles move at v_x but really have distribution of velocities

$$\therefore \quad P = m \frac{N}{V} \langle v_x^2 \rangle$$

$\langle \ \rangle$ means average

v_x not so useful!

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$\therefore \quad \langle v_x^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

since all directions equal

$$\Rightarrow \quad P = \frac{1}{3} m \frac{N}{V} \langle v^2 \rangle$$

$$\Rightarrow \quad \boxed{P = \frac{2}{3} \frac{N}{V} \left\langle \frac{1}{2} m v^2 \right\rangle}$$

$$\boxed{PV = \frac{2}{3} N \left\langle \frac{1}{2} m v^2 \right\rangle}$$

Average KE of a particle

FOR A MONATOMIC GAS

Only energy is translational KE
- no rotational energy
- no vibrational energy

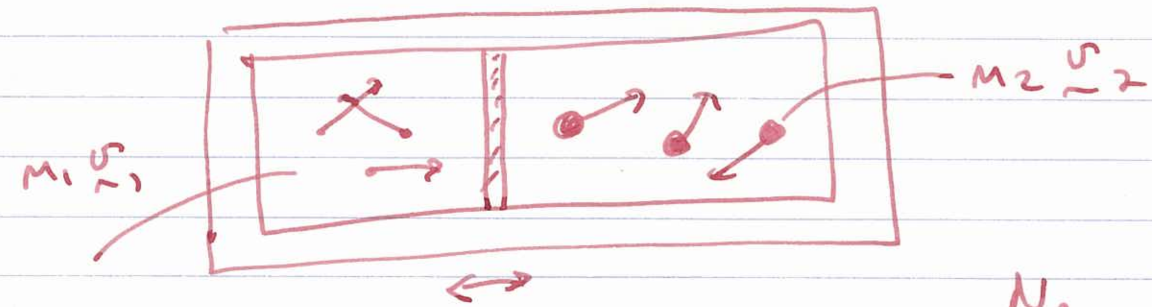
$\therefore$ Internal Energy:

$$U = N \left\langle \frac{1}{2} m v^2 \right\rangle$$

$$\Rightarrow \boxed{PV = \frac{2}{3} U}$$

- looking like an equation of state!

MECHANICAL EQM



$$n_1 = \frac{N_1}{V_1} \qquad n_2 = \frac{N_2}{V_2}$$

At mechanical EQM:

$$P_1 = \frac{2}{3}\frac{N_1}{V_1}\left\langle \frac{1}{2} m_1 v_1^2 \right\rangle$$

$$P_2 = \frac{2}{3}\frac{N_2}{V_2}\left\langle \frac{1}{2} m_2 v_2^2 \right\rangle$$

At ~~eq~~ mechanical eqm

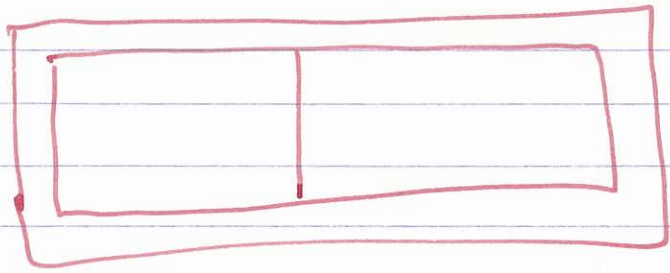
$$\Rightarrow \quad P_1 = P_2$$

$$\Rightarrow \quad \frac{N_1}{V_1}\left\langle \frac{1}{2} m_1 v_1^2 \right\rangle = \frac{N_2}{V_2}\left\langle \frac{1}{2} m_2 v_2^2 \right\rangle$$

$$\Rightarrow \quad n_1 \left\langle \frac{1}{2} m_1 v_1^2 \right\rangle = n_2 \left\langle \frac{1}{2} m_2 v_2^2 \right\rangle$$

TEMPERATURE

Repeat with a thin $\begin{Bmatrix} \text{Rigid} \\ \text{metal} \end{Bmatrix}$ wall



① 1st see mechanical eqm

$$n_1 \left\langle \tfrac{1}{2} M_1 v_1^2 \right\rangle = n_2 \left\langle \tfrac{1}{2} M_2 v_2^2 \right\rangle$$

Then see 2nd EqM:

$$\left\langle \tfrac{1}{2} M_1 v_1^2 \right\rangle = \left\langle \tfrac{1}{2} M_2 v_2^2 \right\rangle$$

Why?

Look at Example:

Assume $M_1 = M_2$ - same atoms

Assume left is dense

& right refractory

$\Rightarrow$ $n_1 \gg n_2$ say $n_1 = 10 n_2$

$\therefore$ at MECHANICAL EQM:

$$\frac{1}{2} m\, n_1 \langle v_1^2 \rangle = \frac{1}{2} m \frac{n_1}{10} \langle v_2^2 \rangle$$

$$\Rightarrow \quad 10 \langle v_1^2 \rangle = \langle v_2^2 \rangle$$

So from left: 10 x as many hits on wall

From right: each hit has 10 x as much energy

$\Rightarrow$ collisions will no longer be elastic, & energy will transfer from Right (big hits) to left

Energy transfer process stops when

$$\langle \tfrac{1}{2} M_1 v_1^2 \rangle = \langle \tfrac{1}{2} M_2 v_2^2 \rangle$$

"Thermal Eqm"