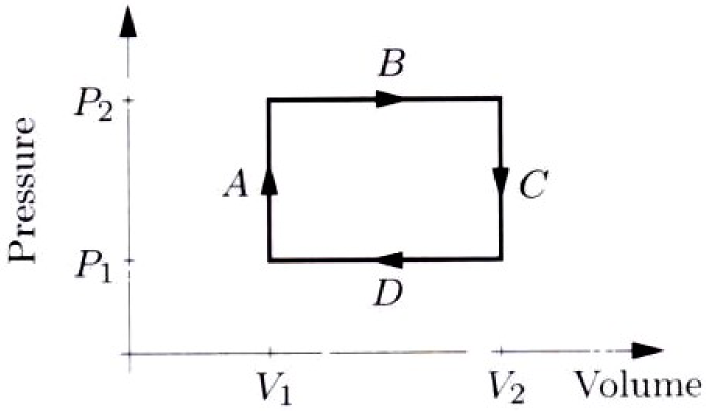
PHYS2060 – Thermal Physics
Session 2 - 2007
Tutorial Problems – Set 3 (Weeks 6-8)

These problems are intended to illustrate and reinforce the course material. It is important to work through these problems, or attempt to. Some will be done in class as examples. Problems marked (*) are more difficult and are included as a challenge, but should be attempted if possible as they will best test your knowledge/skills.

1. Calculate the total internal energy in a litre of helium gas at room temperature and atmospheric pressure. Repeat the calculation for air assuming $\gamma = 1.4$ for air.
2. A voltage is connected in series to a resistor, which is immersed in water (this is exactly how your household kettle works). Would you classify the flow of energy from the battery to the resistor as "heat" or "work"? What about the flow of energy from the resistor to the water?
3. An ideal diatomic gas, in a cylinder with a moveable piston, undergoes the rectangular cyclic process shown below. Assume that the temperature is always such that the rotational degrees of freedom are active, but the vibrational modes are 'frozen out'. Also assume that the only type of work done on the gas is quasistatic compression-expansion work.



   (a) For each of the four steps A through D, compute the work done on the gas, the heat added to the gas, and the change in the internal energy of the gas. Express all answers in terms of P_1, P_2, V_1 and V_2. (Hint: Compute ΔU before Q, using the ideal gas law and the equipartition theorem.)

   (b) Describe in words what is physically being done during each of the four steps; for example, during step A, heat is added to the gas (from an external flame or something) while the piston is held fixed.

   (c) Compute the net work done on the gas, the net heat added to the gas, and the net change in the energy of the gas during the entire cycle. Are the results as you expected? Explain briefly.

4. To measure the heat capacity of an object, all you usually have to do is put it in thermal contact with another object whose heat capacity you know. As an example, suppose that a chunk of metal is immersed in boiling water (100°C), then is quickly transferred into a Styrofoam cup containing 250g of water at 20°C. After a minute or so the temperature

of the contents of the cup is 24°C. Assume that during this time no significant energy is transferred between the contents of the cup and the surroundings. The heat capacity of the cup is negligible.

(a) How much heat is gained by the water?

(b) How much heat is lost by the metal?

(c) What is the heat capacity of this chunk of metal?

(d) If the mass of the chunk of metal is 100g, what is its specific heat capacity?

5. When spring arrives in the mountains, the snow pack is may be two meters deep (except this year which is the worst season in over 20 years ☹), composed of 50% ice and 50% air. Direct sunlight provides about 1000 watts/m^2 to the Earth's surface, but the snow might reflect 90% of this energy. Roughly estimate how many weeks the snow pack should last, if direct solar radiation is the only source of energy. (Hint: think carefully about any approximations and corrections to the normal logic you'd need to make here).

6. Geologists measure conductive heat flow out of the earth by drilling holes (a few hundred meters deep) and measuring the temperature as a function of depth. Suppose that in a certain location the temperature increases by 20°C per kilometre of depth and the thermal conductivity of the rock is 2.5 W/mK. What is the rate of heat conduction per square meter in this location? Assuming that this value is typical of other locations over all of the earth's surface, at approximately what rate is the earth losing heat via conduction? (The radius of the earth is 6400 km).

7. At what pressure would the mean free path of an air molecule at room temperature equal 10cm, the size of a typical laboratory apparatus?

8. (*) Imagine a narrow pipe, filled with fluid, in which the concentration of a certain type of molecule varies only along the length of the pipe (in the *x* direction). By considering the flux of these particles from both directions into a short segment *dx*, derive Fick's second law:

$$\frac{dn}{dt} = D\frac{d^2n}{dx^2}$$

9. (**) In analogy with the thermal conductivity, derive an approximate formula for the diffusion coefficient of an ideal gas in terms of the mean free path and the average thermal speed. Evaluate your formula numerically for air at room temperature and atmospheric pressure, and compare to the experimental value of 2×10^{-5} m^2/s. How does D depend on T at fixed pressure?

10. Consider the cycle in question 3, which involved an ideal diatomic gas taken around a rectangular cycle on a PV diagram. Suppose now that this system is used as a heat engine to convert the heat added into mechanical work

(a) Evaluate the efficiency of this engine for the case $V_2 = 3V_1$, $P_2 = 2P_1$.

(b) (*) Calculate the efficiency of an 'ideal' engine operating between the same temperature extremes.

11. A power plant that produces 1GW (10^9 watts) of electricity, the steam turbines take in steam at a temperature of 500°C, and the waste heat is expelled into the environment at 20°C.

(a) What is the maximum possible efficiency of the plant?

(b) Suppose you develop a new material for making pipes and turbines which allows the maximum steam temperature to be raised to 600°C. Roughly how much money can you make in a year by installing your improved hardware if you sell the additional electricity for 5 cents per kilowatt-hour? (Assume that the amount of fuel consumed at the plant is unchanged).