

PHYS2060 - Tutorial 3

1. He has 3 degrees of freedom (translational) per molecule as its monatomic. So $U = 3N \cdot \frac{1}{2} k_B T = \frac{3}{2} PV$. For $P = 1.01 \times 10^5 \text{ N/m}^2$ and $V = 10^{-3} \text{ m}^3$, the product PV is 101 J, so $U = 151.5 \text{ J}$.

For air, with $\gamma = 1.4$, we have a diatomic molecule with five degrees of freedom (three translational and two rotational) at room T , so $U = \frac{5}{2} PV$ (ie γPV) = 252.5 J.

2. The flow of energy from the battery to the resistor is work not heat. Even though the resistor gets warm, there is no spontaneous flow of energy from the battery to the resistor that is caused by a difference in their temperatures. In fact, the battery could be colder than the resistor and keep providing energy to it. The flow of energy from the resistor to the water, however, is a spontaneous one caused by the resistor being hotter than the water. This energy flow is therefore classified as heat, not work.

3. It's easiest to first compute the work done during each step, using $W = -P\Delta V$. For steps A and C there is no work done because the volume doesn't change. For steps B and D the pressure is constant so we don't need to set up an integral. So, for instance, the work done on the gas during step D is $+P_1(V_2 - V_1)$. Since each molecule has five active degrees of freedom, the thermal energy of the gas at any time is $U = \frac{5}{2} N k_B T = \frac{5}{2} PV$, therefore ΔU during any step is $\frac{5}{2}(P_F V_F - P_i V_i)$, where F stands for final and i for initial. So, during step D, $\Delta U = -\frac{5}{2} P_1(V_2 - V_1)$. The heat added to the gas during any step is just $Q = \Delta U - W$, so again for step D we get $Q = -\frac{5}{2} P_1(V_2 - V_1) - P_1(V_2 - V_1) = -\frac{7}{2} P_1(V_2 - V_1)$.

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3. A table for all four steps computed in this way:

	<u>W</u>	<u>ΔU</u>	<u>Q</u>
step A	0	$\frac{5}{2} V_1 (P_2 - P_1)$	$\frac{5}{2} V_1 (P_2 - P_1)$
step B	$-P_2 (V_2 - V_1)$	$\frac{5}{2} P_2 (V_2 - V_1)$	$\frac{7}{2} P_2 (V_2 - V_1)$
step C	0	$-\frac{5}{2} V_2 (P_2 - P_1)$	$-\frac{5}{2} V_2 (P_2 - P_1)$
step D	$P_1 (V_2 - V_1)$	$-\frac{5}{2} P_1 (V_2 - V_1)$	$-\frac{7}{2} P_1 (V_2 - V_1)$
whole cycle.	$-(P_2 - P_1)(V_2 - V_1)$	0	$(P_2 - P_1)(V_2 - V_1)$

What's actually happening is something like: During step A we hold the piston fixed but put heat in (say from a flame); during step B we let the piston out and continue putting heat in at such a rate as to maintain constant pressure; during step C we hold the piston fixed but suck heat out, perhaps by immersing the whole thing in an ice bath; and during step D we push the piston in while still sucking out heat so the pressure again remains steady.

The net work done on the gas during the whole cycle is negative, in other words, the net work done by the gas is positive (it would make a good engine). This is as expected because the pressure is higher when the gas is expanding than when it is being compressed. Notice that the net work is just minus the area enclosed by the rectangular cycle on the diagram. The net change in the internal energy of the gas is zero, as it must be: the state of the gas (as determined by its pressure and volume) is the same at the end of a cycle as at the beginning. Therefore the net heat put into the gas must be minus the net work done, as indeed it is. In summary, this procedure results in a net conversion of heat input into work output.

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4. (a). The heat gained by the water is

$$m_w c_w (\Delta T)_w = 250 \text{ g} \times 4.186 \text{ J/gK} \times 4 \text{ K} = 4186 \text{ J}$$

(b) The heat lost by the metal must be the same as the heat lost by the water, 4186 J, since there are no other energy transfers going on and energy must be conserved.

(c) The heat capacity of the chunk of metal must therefore be

$$C_m = \frac{Q}{\Delta T_m} = \frac{-4186 \text{ J}}{-76^\circ \text{C}} = 55 \text{ J/}^\circ \text{C}.$$

(d) The specific heat capacity is the heat capacity per unit mass,

$$c_m = \frac{C_m}{m_m} = \frac{55 \text{ J/K}}{100 \text{ g}} = 0.55 \text{ J/gK}.$$

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Q5 **Problem 1.48.** Consider a 1-m^2 patch of snow-covered ground, which is covered by 1 m^3 of ice (and an equal amount of air mixed with it, which we can neglect). That's nearly 1000 kg, so if it's already at 0°C , the energy needed to melt it is 80 million calories or 335 MJ. Now direct sunlight provides to this patch of ground 1000 J/s, but only 10%, or 100 J/s, is absorbed. A full day of sunshine in late spring might be equivalent to 8 hours, or about 30,000 seconds, of direct sunlight. So in one day the snow absorbs about 3 MJ of solar energy. That means it should take more than 100 days, or 14 weeks, for the snow to melt! Probably, though, the snow absorbs more energy from other sources such as the air and any nearby rocks or trees.

Q6 **Problem 1.61.** For the numbers given, the rate of heat conduction should be

$$\frac{Q}{\Delta t} = k_t A \frac{dT}{dx} = (2.5 \text{ W/m}\cdot^\circ\text{C})(1 \text{ m}^2) \frac{20^\circ\text{C}}{1000 \text{ m}} = 0.05 \text{ W}.$$

A twentieth of a watt doesn't sound like much, but that's just for one square meter. The surface area of the whole earth is $4\pi r^2 = 4\pi(6.4 \times 10^6 \text{ m})^2 = 5.1 \times 10^{14} \text{ m}^2$, so the total rate of heat conduction through earth's crust should be 2.6×10^{13} watts, assuming that this particular location is representative of the average.

Q7 **Problem 1.63.** Substituting kT/P for V/N in equation 1.62 for the mean free path ℓ , we have

$$\ell = \frac{1}{4\pi r^2} \frac{kT}{P}, \quad \text{or} \quad P = \frac{1}{2\pi r^2} \frac{kT}{\ell}.$$

Setting $T = 300 \text{ K}$, $r = 1.5 \text{ \AA}$, and $\ell = 10 \text{ cm}$, I find

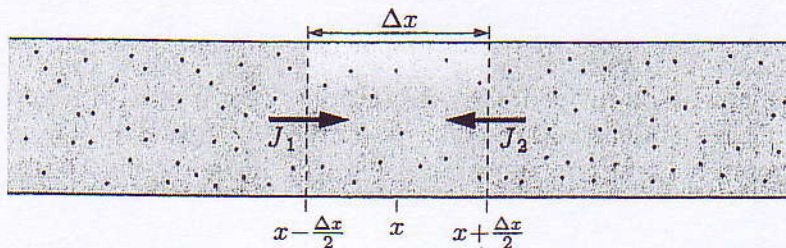
$$P = \frac{1}{4\pi(1.5 \times 10^{-10} \text{ m})^2} \frac{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{0.1 \text{ m}} = 0.15 \text{ Pa} = 1.5 \times 10^{-6} \text{ atm}.$$

That's well within the range of a good mechanical pump, and nowhere near the state of the art in laboratory vacuum systems. Therefore, in a good laboratory vacuum system, molecules will collide with the walls of the apparatus much more often than they collide with each other.

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Q 8

Problem 1.69. Consider the segment of the pipe shown below, with fluxes J_1 and J_2 coming in from the left and right:



Since flux is the number of particles crossing a surface per units area per unit time, the rate of change of the number of particles within the segment is equal to the total flux times the cross-sectional area:

$$\frac{dN}{dt} = (J_1 + J_2) \cdot A.$$

Dividing by the volume of the segment gives the rate of change of the number density:

$$\frac{dn}{dt} = \frac{dN/dt}{A \Delta x} = \frac{1}{\Delta x} (J_1 + J_2).$$

Now express each flux in terms of dn/dx using Fick's (first) law, being careful with the signs:

$$\frac{dn}{dt} = \frac{1}{\Delta x} \left(-D \frac{dn}{dx} \Big|_{x-\frac{\Delta x}{2}} + D \frac{dn}{dx} \Big|_{x+\frac{\Delta x}{2}} \right).$$

The right-hand side is just D times the second derivative of n with respect to x , so we finally arrive at the partial differential equation

$$\frac{dn}{dt} = D \frac{d^2 n}{dx^2}.$$

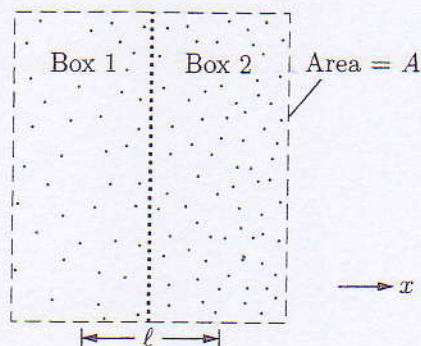
This equation has exactly the same form as the heat equation derived in Problem 1.62, with temperature here replaced by n and the constant K here replaced by D . Therefore particle diffusion behaves in the same way (mathematically) as heat conduction, and any solution of the heat equation can be carried over immediately to the case of diffusion. In particular, the spreading Gaussian solution discussed in Problem 1.62 becomes

$$n(x, t) = n_0 + \frac{a}{\sqrt{t}} e^{-x^2/4Dt},$$

representing an initially localized concentration of particles (above the background concentration n_0) that spreads out quickly at first and more slowly later on.

Q9

Problem 1.70. Consider two "boxes" within the gas, each of width ℓ (the mean free path) and cross-sectional area A :



During a time interval Δt equal to the average time between collisions (ℓ/\bar{v}), about half the molecules in Box 1 will move to Box 2, while about half the molecules in Box 2 move to Box 1. Denoting the concentrations of molecules within the boxes as n_1 and n_2 , the net migration of molecules from left to right is then roughly

$$\frac{1}{2}n_1 A \ell - \frac{1}{2}n_2 A \ell = \frac{1}{2}A \ell (n_1 - n_2),$$

and the flux is this quantity divided by A and by Δt :

$$J_x \approx \frac{A \ell (n_1 - n_2)}{2A \Delta t} = \frac{\ell (n_1 - n_2)}{2\ell/\bar{v}} = -\frac{1}{2}\ell \bar{v} \frac{dn}{dx}.$$

Comparing to Fick's law (equation 1.70), we can read off the diffusion coefficient as approximately $\frac{1}{2}\ell \bar{v}$. For air at room temperature, this evaluates to

$$D \approx \frac{1}{2}\ell \bar{v} = \frac{1}{2}(1.5 \times 10^{-7} \text{ m})(500 \text{ m/s}) = 4 \times 10^{-5} \text{ m}^2/\text{s},$$

about twice the experimental value quoted in the text. Since ℓ is proportional to $V/N = kT/P$ while \bar{v} is proportional to \sqrt{T} , the diffusion coefficient should be proportional to $T^{3/2}/P$. Therefore, at fixed pressure, doubling the temperature should increase the diffusion coefficient by a factor of about 2.8.

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Q10. **Problem 4.1.** (Ideal gas engine with rectangular PV cycle.)

(a) The net work done by the gas during one cycle is

$$|W| = (P_2 - P_1)(V_2 - V_1) = (P_1)(2V_1) = 2P_1V_1,$$

while the heat absorbed (during steps A and B) is

$$Q_h = \frac{5}{2}V_1(P_2 - P_1) + \frac{7}{2}P_2(V_2 - V_1) = \frac{5}{2}V_1P_1 + 14P_1V_1 = \frac{33}{2}P_1V_1.$$

Therefore the efficiency is

$$e = \frac{|W|}{Q_h} = \frac{2P_1V_1}{\frac{33}{2}P_1V_1} = \frac{4}{33} = 12\%.$$

(b) The relative temperatures at various points around the cycle can be determined from the ideal gas law, $PV = NkT$. The lowest temperature occurs at the bottom-left corner when P and V are both smallest. As the pressure doubles during step A the temperature also doubles; then as the volume is tripled during step B so is the temperature. Thus the highest temperature, at the upper-right corner, is six times as great as the lowest temperature. For these extreme temperatures the maximum possible efficiency would be

$$e_{\max} = 1 - \frac{T_c}{T_h} = 1 - \frac{T_c}{6T_c} = \frac{5}{6} = 83\%.$$

The rectangular cycle is extremely inefficient compared to a Carnot cycle.

Q11. **Problem 4.2.** (A steam power plant.)

(a) For these extreme temperatures the maximum efficiency would be

$$e = 1 - \frac{T_c}{T_h} = 1 - \frac{293 \text{ K}}{773 \text{ K}} = 62.1\%.$$

(b) With the higher steam temperature the maximum efficiency would be

$$e = 1 - \frac{T_c}{T_h} = 1 - \frac{293 \text{ K}}{873 \text{ K}} = 66.4\%.$$

If this efficiency is actually attained, then for a given Q_h we would get more work output than before by a factor of

$$\frac{.664}{.621} = 1.069,$$

that is, we get an additional .069 GW of power. To compute the additional profit, multiply the extra energy by the price charged:

$$\Delta\$ = (.069 \times 10^9 \text{ J/s})(3.16 \times 10^7 \text{ s/yr}) \left(\frac{1 \text{ kw-hr}}{3.6 \times 10^6 \text{ J}} \right) \left(\frac{.05 \$}{1 \text{ kw-hr}} \right) = 3 \times 10^7 \$.$$

Not bad: we make 30 megabucks!