PHYS2060 – Thermal Physics
Session 2 - 2007
Tutorial Problems – Set 1 (Weeks 1-3)

These problems are intended to illustrate and reinforce the course material. It is important to work through these problems, or attempt to. Some will be done in class as examples. Problems marked (*) are more difficult and are included as a challenge, but should be attempted if possible as they will best test your knowledge/skills.

1. The Fahrenheit temperature scale is defined so that ice melts at 32°F (0°C) and water boils at 212°F (100°C).
   (a) Derive the formulas for converting from Fahrenheit to Celcius and back.
   (b) What is absolute zero in the Fahrenheit scale?

2. Give an example to illustrate why you cannot accurately judge the temperature of an object by how hot or cold it feels to the touch. Why is this so?

3. Estimate the number of air molecules in an average sized room (hint: an average sized room is 5m × 5m × 2m). How much does this air weigh?

4. Calculate the average volume per molecule for an ideal gas at room temperature and atmospheric pressure. Then take the cube root to get an estimate of the average distance between molecules. How does this distance compare to the size of a small molecule like N_2 or H_2O? (n.b. you will need to look these up)

5. From equipartition of energy you know that $\langle \frac{1}{2}mv^2 \rangle = 3/2 k_B T$, derive and expression for the root-mean-square speed of a gas molecule.

6. Uranium has two common isotopes, with atomic masses of 238 and 235. One way to separate these isotopes is to combine the uranium with fluorine to make uranium hexafluoride (UF_6) gas, and then exploit the difference in the average thermal speeds of molecules containing the different isotopes. Calculate the rms speed of each type of molecule at room temperature, and compare them. (n.b., you will need to do question 5 first).

7. (*) List all the degrees of freedom, or as many as you can, for a molecule of water vapour. What would you expect the specific heat C vs temperature T for water vapour to look like? (n.b. you will be able to give quantitative values for the C-axis, but not the T axis)

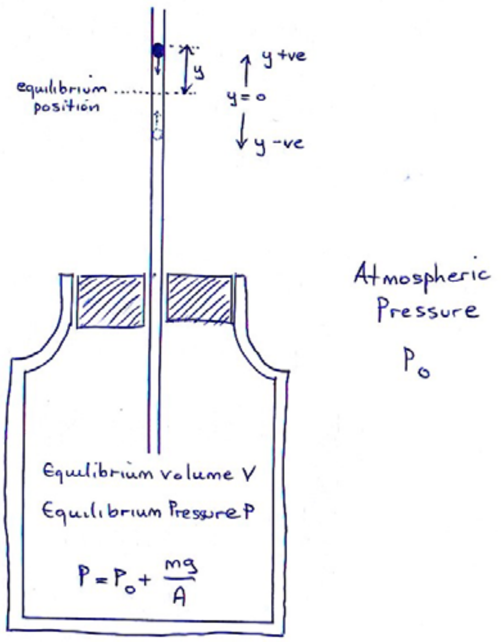
   (hint: think carefully about all the various ways in which the molecule can rotate and vibrate).

8. In the course of pumping up a bicycle tire, a litre of air at atmospheric pressure is compressed adiabatically to a pressure of 7 atm.

   (a) What is the final volume of this air after compression?

   (b) If $PV^{\gamma} = C_0$, show that $TV^{\gamma-1} = C_1$ where $C_0 \neq C_1$

   (c) If the temperature of the air is initially 300K, what is the temperature after compression?



9. (*) An ingenious method for measuring γ was developed by Rüchhardt in 1929. The apparatus is shown above and consists of a vessel of volume V with a narrow tube passing vertically out the top. It is filled and surrounded by the gas of interest with a pressure P_0 The tube has an accurate bore with a cross-sectional area A, into which a metal ball of mass m fits snugly like a piston. At its equilibrium position, the mass of the ball compresses the gas in the vessel slightly so that the pressure of the gas P inside the vessel is slightly larger than the pressure P_0 outside such that:

$$P = P_0 + \frac{mg}{A}$$

If the ball is given a slight downward displacement and then let go, it will oscillate with period τ. Let the ball's displacement from its equilibrium position be y and assume that the gas behaves adiabatically (i.e., you can use PV^{γ} = const). Show that the adiabatic parameter γ is given by:

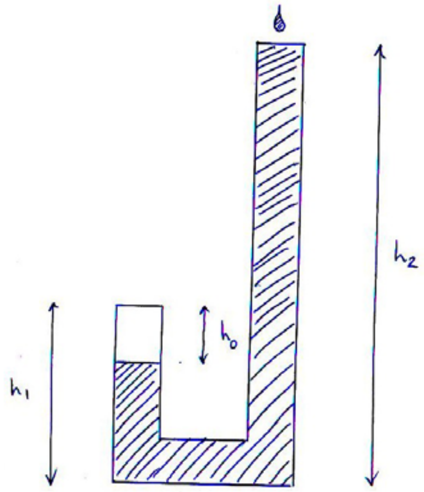
$$\gamma = \frac{4\pi^2 mV}{A^2 P\tau^2}$$

10. (*) Suppose we have a gas of molecules that move with a speed comparable to the speed of light c. (Suppose also that the molecules survive their collision with the wall!)
    (a) Adapt the kinetic theory analysis to this context; express the pressure in terms of the rest mass m and the relativistic energy $\varepsilon_{rel} = mc^2/\sqrt{1 - v^2/c^2}$ and eliminate the speed v entirely.
    (b) After giving the general expression, examine the domain in which the strong inequality $\langle \varepsilon_{rel} \rangle >> mc^2$ holds. Compare this limit with the pressure result for a photon gas $P = \frac{1}{3}\frac{N}{V}\langle \varepsilon_{ph} \rangle$ where $\langle \varepsilon_{ph} \rangle$ is the photon energy.

11. Estimate the pressure (in atm) exerted on the floor by an average person wearing (a) thongs and (b) stilleto heels. (n.b., you will need to come up with reasonable estimates for the body weight and sole contact area in each case to complete this question).

12. A mixture of hydrogen and oxygen is isolated and allowed to reach a state of constant temperature and pressure. The mixture is then exploded with a spark of negligible energy and allowed to come to a state of constant temperature and pressure. (a) Is the initial state an equilibrium state, and if so, what kind? Explain. (b) Is the final state an equilibrium state, and if so, what kind? Explain.

13. [n.b. this isn't the greatest question] - (*) A J-shaped tube of uniform cross-sectional area *A*, as shown below, contains air at atmospheric pressure. Mercury is poured into the open end, trapping the air in the closed end. What is the height *h* of the mercury column in the closed end, when the open end is filled to the brim with mercury? Assume that the temperature is constant, the density of mercury is 13.5 g/cm^3 and that air is an ideal gas. Neglect any effect of curvature at the bottom. As a numerical example, let h_1 = 0.25m and h_2 = 2.25m, calculate *h* and the pressure *P* of the gas in the closed end.



14. A tank of volume 0.5m^3 contains oxygen at an absolute pressure of 1.5 x 10^6 N/m^2 and a temperature of 20°C. Assume that oxygen behaves like an ideal gas. (a) How many moles of oxygen are there in the tank? (b) How many kilograms? (c) Find the pressure if the temperature is increased to 500°C. (d) At a temperature of 20°C, how many moles can be withdrawn from the tank before the pressure falls to 10% of the original pressure?

15. Explain how it follows from the 0th law of thermodynamics that temperature is a universal property: one which is possessed in common by all systems whatever their nature.

16. A toy hot air balloon consists of a spherical envelope 1.5m in diameter below which is suspended a burner. Air heated by the burner enters the balloon through an opening at the bottom and collects in the envelope. If the total mass is 0.3kg, how hot does the air in the envelope become before the balloon rises? (Ignore any change in composition of the air due to products of combustion and take the ambient temperature as 15°C. Density of air at s.t.p. = 1.29 kg/m^3)

17. (**) Explain how Joule's law follows from the fact that an ideal gas does not change its temperature when it makes a free expansion. Show how it follows from Joule's law that changes in the internal energy of an ideal gas may always be written $dU = C_v dT$. (n.b., the second part of this question relies on use of the 1st law.)

18. Two vessels in thermal contact are connected by a tube with a tap in it. The first vessel has a volume of 2×10^{-3} m^3 and contains an ideal gas at a pressure of 2 atm. The second vessel has a volume of 3×10^{-3} m^3 and is evacuated. The tap is opened. When the system has reached equilibrium,

    (a) What is the temperature change?

    (b) What is the pressure?

    (c) What is the ratio of the masses of gas in the two vessels?

    (d) What essential difference would it make if the vessels were not in thermal contact?

    [hint: remember that the total number of moles is a constant].

19. A diesel engine takes in atmospheric air and compresses it adiabatically so that it becomes hot. Fuel is then injected into the compressed air and the fuel-air mixture spontaneously ignites (i.e., no spark plug) provided that the temperature is high enough. Work is obtained as the resulting high pressure gases force back the piston. If the auto-ignition temperature of diesel fuel is 430°C and the air enters the engine at 17°C, what is the minimum compression ratio (ratio of initial volume to compressed volume) which must be used? [for air $\gamma = 1.40$]

20. (**) Why does pressure increase with depth linearly (i.e., directly proportional to) in a lake, but exponentially in the atmosphere?