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1(a) Far above the Earth's surface, the temperature is ~1000K, but the pressure and air density are also extremely low. The heat in the air is transferred to the astronaut by conduction when air molecules hit the astronauts' surface, but these collisions are quite rare because of the rarefied atmosphere. At the same time, the astronaut is radiating energy as a blackbody, and losing heat. This heat loss is much faster than the heat gain, and so ultimately the astronaut will freeze to death rather than vaporize.

1(b). Temperature is the quantity that is equal in two bodies when they are at thermal equilibrium

Heat is energy that is spontaneously transferred between two bodies that have different temperatures

Internal energy is the sum of the energies of all of the microscopic components of a gas, for a monatomic gas, $U = N\langle \frac{1}{2}mv^2 \rangle$ where N is the number of particles and $\langle \frac{1}{2}mv^2 \rangle$ is their average kinetic energy.

Temperature and internal energy are related by the principle of equipartition of energy such that $U = \frac{3}{2}Nk_BT$. The change in internal energy ΔU is related to Q by the first law $\Delta U = Q + W$. Q is roughly related to the difference in temperature ΔT between two bodies, but this is complicated.

1(c). Yes, as long as they have the same temperature they are at thermal equilibrium they don't need to be in contact. If they have different temperatures, they can equilibrate without being brought into contact also, either by conduction through another medium or by radiation

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2(a) There is no heat transferred between the system and its surroundings during the process. Adiabatic processes can be achieved by performing them very quickly or by heavily insulating the system from its surroundings.

2(b) $$W = -\int_{V_i}^{V_F} P\,dV = -\int_{V_i}^{V_F} \frac{C}{V^\gamma}\,dV = -\int_{V_i}^{V_F} CV^{-\gamma}\,dV$$

$$= -\left[\frac{C}{1-\gamma} V^{1-\gamma}\right]_{V_i}^{V_F}$$

$$= \frac{C}{\gamma - 1}\left(\frac{V_F}{V_F^\gamma} - \frac{V_i}{V_i^\gamma}\right)$$

$$= \frac{1}{\gamma - 1}\left(P_F V_F - P_i V_i\right)$$ as required

2(c) The initial volume of air is 12cm^3, the final volume is $12 + Ah = 12 + 0.03 \times 50 = 13.5\text{cm}^3$. The final pressure P_F and $P_F V_F^\gamma = P_i V_i^\gamma$, so $P_F = P_i \left(\frac{V_i}{V_F}\right)^\gamma$

When the bullet leaves the rifle it has an energy $\frac{1}{2}mv^2 = \frac{1}{2} \times 0.0011 \times 120^2 = 7.92\,\text{J}$. By the first law/conservation of energy, this energy must be supplied by the work done by the expanding gas.

$$\therefore \quad W = \frac{1}{\gamma - 1}\left(P_F V_F - P_i V_i\right) = 7.92\,\text{J}$$ and we're after P_i

$$\text{so} \quad -7.92\,\text{J} = \frac{1}{0.4} P_i \left[V_F \left(\frac{V_i}{V_F}\right)^\gamma - V_i\right]$$

the - because the gas does work on the bullet.

$$\therefore \quad P_i = \frac{-7.92\,\text{J} \times 0.4}{\left[1.35 \times 10^{-5} \left(\frac{1.2 \times 10^{-5}}{1.35 \times 10^{-5}}\right)^{1.4} - 1.2 \times 10^{-5}\right]}$$

$$= 5.74 \times 10^6\,\text{Pa}$$

$$= 56.6\,\text{atm}.$$

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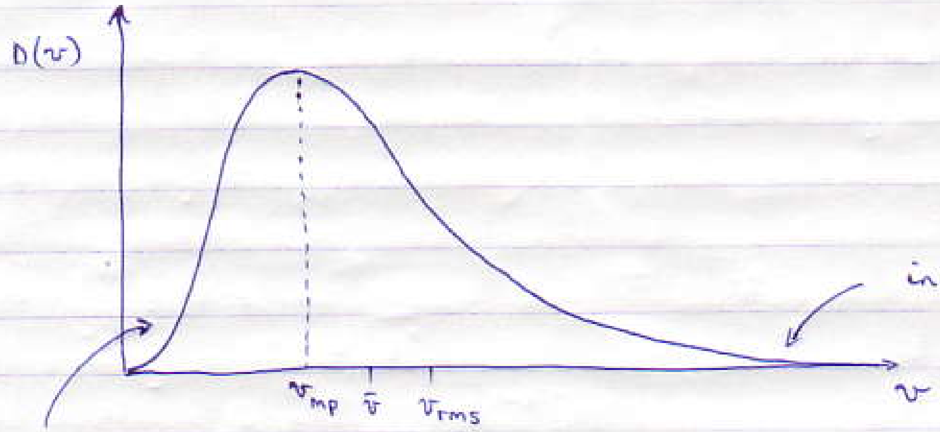
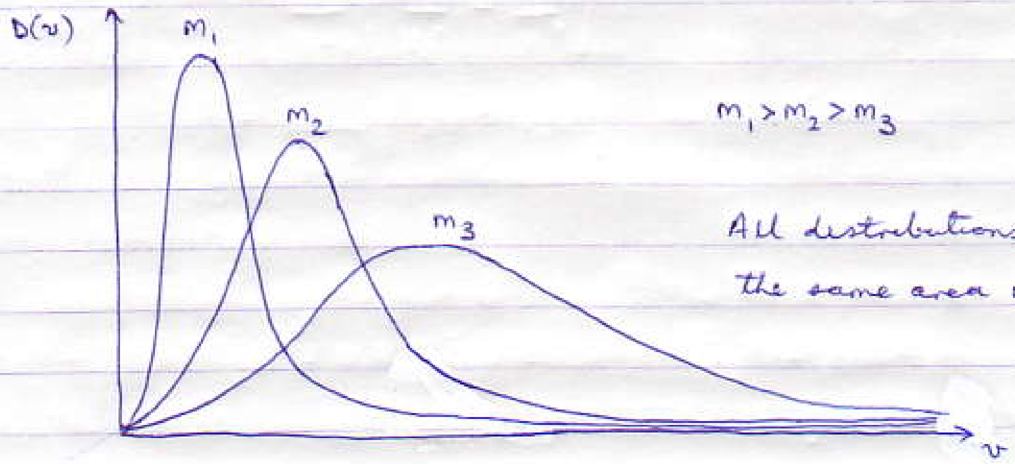
3(a)



in limit of large v
$D(v) \sim \exp(-v^2)$
(exponential)

in limit of small v
$D(v) \sim v^2$
(parabolic)

3(b)



$m_1 > m_2 > m_3$

All distributions have the same area under the curve.

At some constant T, higher particle mass means lower velocity as $mv^2 \sim k_B T$, therefore the leftmost distribution is the highest mass and the rightmost is the lowest mass. Because $D(v)$ is a probability density the area under all curves above must have the same value, specifically $\int_0^\infty D(v)\,dv = 1$.

3(c) $P(v_1 \ldots v_2) = \int_{v_1}^{v_2} D(v)\,dv$. $D(v)$ is a probability density, it needs to be integrated to obtain a probability

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3(d). The most probable speed of a gas molecule corresponds to the peak of $D(v)$, where $\frac{dD(v)}{dv} = 0$. To do this derivative we need to use the chain rule $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

$$D(v) = C_o v^2 \exp\left(\frac{-mv^2}{2k_BT}\right) \qquad \text{where } C_o = \left(\frac{m}{2\pi k_BT}\right)^{3/2} \times 4\pi$$

$$\frac{d(D(v))}{dv} = 2vC_o \exp\left(-\frac{mv^2}{2k_BT}\right) + -\frac{2mv}{2k_BT} C_o v^2 \exp\left(-\frac{mv^2}{2k_BT}\right) = 0$$

$$2v\cancel{C_o}\cancel{\exp\left(\frac{-mv^2}{2k_BT}\right)} = \frac{2mv}{2k_BT}\cancel{C_o} v^2 \cancel{\exp\left(\frac{-mv^2}{2k_BT}\right)}$$

$$2v = \frac{\cancel{2}mv}{\cancel{2}k_BT} v^2$$

$$\frac{2k_BT}{m} = v^2$$

$$v = \sqrt{\frac{2k_BT}{m}} \quad \text{as required}$$

3(e) i. $\frac{1}{2}mv^2 = \frac{3}{2}k_BT$

$$v^2 = \frac{3/2\, k_BT}{1/2\, m} = \frac{3k_BT}{m}$$

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

ii. $PV = Nk_BT \Rightarrow k_BT = \frac{PV}{N}$ divide both sides by mass

$$\frac{k_BT}{m} = \frac{PV}{Nm} = \frac{P}{\rho}$$

Therefore $v_{rms} = \sqrt{\frac{3P}{\rho}}$ as required