

PHYS 2060 - Midsession exam

1 a. i. In time  $t$ , particles within a displacement of  $s = v_x t$  of the piston  
1 mark will hit it. This will give a capture volume  $V = v_x t A$  and with  
 $n = N/V$  particles per volume, this will give  $n_c$  particles striking  
the piston where  $n_c = n v_x t A$

ii. In its collision with the piston,  $v_y$  remains constant, but  $v_x \rightarrow -v_x$ ,  
1 mark in other words the particle bounces elastically reversing its direction,  
as required by conservation of momentum. The change in momentum  
is  $\Delta p = p_f - p_i = -m v_x - m v_x = -2m v_x$  per particle, hence  $\frac{dp}{dn_c} = -2m v_x$ .

1 b. The force on the piston is  $F = \frac{dp}{dt} = \frac{dp}{dn_c} \cdot \frac{dn_c}{dt}$   
1 mark 
$$= 2m v_x \cdot \frac{d}{dt} (n v_x t A)$$
$$= 2m v_x \cdot n v_x A$$

Pressure is force per A  $\therefore P = \frac{F}{A} = 2mn v_x^2$ .

C (i)  $v_x$  differs from particle to particle in the gas, so the natural thing  
1 mark to do is to take the average. But  $\langle v_x \rangle \sim 0$  because although the  
magnitude isn't zero the  $+x$  - directions average each other out.  
using the root-mean-square velocity  $v_{rms} = \langle v_x^2 \rangle^{1/2}$  over comes this  
problem, so  $v_x^2$  can be replaced with  $\langle v_x^2 \rangle$ . The factor of  $1/2$   
accounts for the fact that only half of the particles will be heading right  
and hit the piston, the other half won't.

(ii)  $\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$  but if the gas is isotropic, which it  
1 mark  $= \langle v_x^2 \rangle + \langle v_x^2 \rangle + \langle v_x^2 \rangle$  usually is, then no direction is  
 $= 3 \langle v_x^2 \rangle$  special and so  $\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$   
 $\therefore \langle v_x^2 \rangle = \frac{1}{3} \langle v^2 \rangle$

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1c iii  
1 mark

$$P = 2mn v_{xc}^2 \quad \text{using } v_{xc}^2 \rightarrow \frac{1}{2} \langle v_{xc}^2 \rangle$$
$$P = mn \langle v_{xc}^2 \rangle \quad \text{using } \langle v_{xc}^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$
$$= \frac{mn}{3} \langle v^2 \rangle \quad \text{using } n = \frac{N}{V} \text{ and } \frac{1}{3} = \frac{1}{2} \times \frac{2}{3}$$

$$P = \frac{N}{V} \frac{2}{3} \langle \frac{1}{2} m v^2 \rangle$$

$$PV = \frac{2}{3} N \langle \frac{1}{2} m v^2 \rangle$$

1d.  
1 mark

$$\langle \frac{1}{2} m v^2 \rangle = \frac{3PV}{2N} = \frac{3 \times 12 \times 1.01 \times 10^5 \times 2}{2 \times 6.022 \times 10^{23}} = 6.03 \times 10^{-18} \text{ J (37.7 eV)}$$

1e.  
1 mark

$$\langle \frac{1}{2} m v^2 \rangle = \frac{3}{2} k_B T \Rightarrow T = \frac{2}{3 k_B} \langle \frac{1}{2} m v^2 \rangle$$
$$= \frac{2 \times 6.03 \times 10^{-18}}{3 \times 1.38 \times 10^{-23}}$$
$$= 291684 \text{ K}$$

(student might cheat & use ideal gas law  $T = \frac{PV}{nR} = \frac{12 \times 1.01 \times 10^5 \times 2}{1 \times 8.314} = 291542 \text{ K}$ )

1f. The internal energy  $U$  of a system is all of the energy of a system that is associated with its microscopic components - eg. atoms and molecules - when viewed from a reference frame at rest with respect to the center of mass of the system. A monatomic gas has only three degrees of freedom which are translational motion in the three spatial dimensions, there are no other degrees of freedoms such as rotational or vibrational states.

Hence  $U = N \times \langle \frac{1}{2} m v^2 \rangle$  (number of particles  $\times$  average kinetic energy per particle)

So if  $PV = \frac{2}{3} N \langle \frac{1}{2} m v^2 \rangle$  then  $PV = \frac{2}{3} U$  for a monatomic gas.

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2a. Boyle's law relates  $P$  &  $V$  at const.  $T \Rightarrow P_F V_F = P_i V_i$   
2marks Charles' law relates  $V$  &  $T$  at const.  $P \Rightarrow \frac{V_F}{T_F} = \frac{V_i}{T_i}$   
(he was the balloon guy 😊).

i. Taking the third law as const  $V$  then Boyle's gives  $P_F = P_i$   
and Charles' gives  $\frac{1}{T_F} = \frac{1}{T_i}$

$\therefore$  the third law should be  $\frac{P_F}{T_F} = \frac{P_i}{T_i}$  @ constant  $V$ .

ii. Looking at the three, a combined gas law should be  $\frac{P_F V_F}{T_F} = \frac{P_i V_i}{T_i}$

@ const  $T \Rightarrow P_F V_F = P_i V_i$  = Boyle's law

@ const  $P \Rightarrow \frac{V_F}{T_F} = \frac{V_i}{T_i}$  = Charles' law.

@ const  $V \Rightarrow \frac{P_F}{T_F} = \frac{P_i}{T_i}$  = Third law so this is right.

2b. Joule's law states that the internal energy of an ideal gas depends  
2marks only on its temperature. The combined gas law can also  
be written as  $\frac{PV}{T} = \text{constant}$ , and the ideal gas law says  
that this constant =  $nR$  where  $n$  is the number of moles and  
 $R$  is the ideal gas constant, in other words  $PV = nRT$ .

Comparing with the kinetic expression  $PV = \frac{2}{3}U$ , we'd get

$U = \frac{3}{2}nRT$ , which agrees with Joule's law (n.b. in general  $U = nRT(\frac{5}{2})$ )

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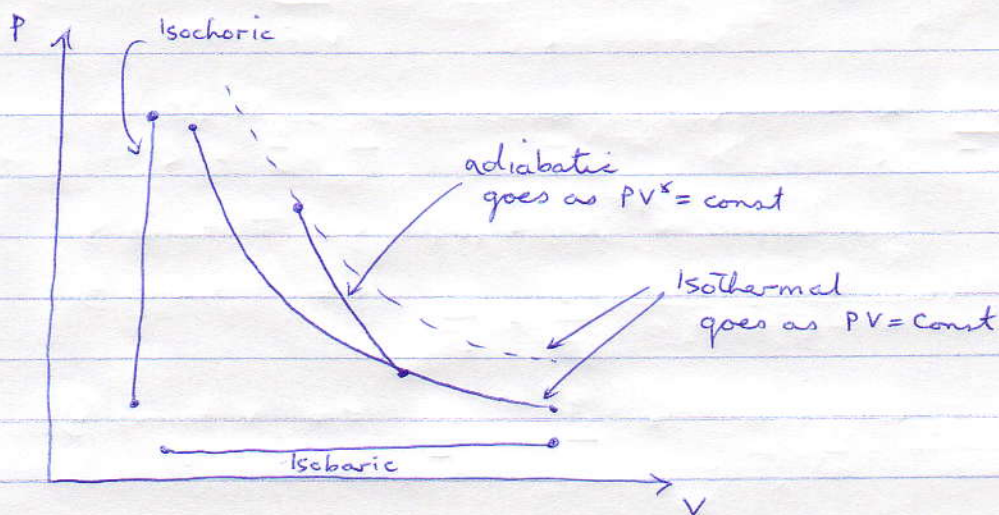
2c. For an adiabatic process  $PV^\gamma = C_0$ , but we also have the  
2marks combined gas law  $\frac{PV}{T} = \text{const}$ , lets call this  $C_1$   $\therefore P = \frac{C_1 T}{V}$

$$\text{So } PV^\gamma = C_0 \Rightarrow \frac{C_1 T V^\gamma}{V} = C_0$$

$$T \frac{V^\gamma}{V} = \frac{C_0}{C_1} = C_2$$

$$TV^{\gamma-1} = \text{const} = C_2 \quad \text{w/ } C_2 \neq C_1 \neq C_0.$$

2d.  
2marks



An adiabatic process will run between the isotherms corresponding to the initial and final temperatures of the process.

2e. i. Adiabatic case:  $P_F V_F^\gamma = P_i V_i^\gamma$   
2marks 
$$P_F = \frac{P_i V_i^\gamma}{V_F^\gamma} = P_i \left( \frac{V_i}{V_F} \right)^\gamma = 1 \left( \frac{10}{2} \right)^{1.4} = 9.5 \text{ atm.}$$

isothermal case:  $P_F V_F = P_i V_i$   
$$P_F = \frac{P_i V_i}{V_F} = P_i \left( \frac{V_i}{V_F} \right) = 1 \left( \frac{10}{2} \right) = 5 \text{ atm.}$$

ii  $T_F V_F^{\gamma-1} = T_i V_i^{\gamma-1} \Rightarrow T_F = T_i \left( \frac{V_i}{V_F} \right)^{\gamma-1} = 298 \text{ K} \times \left( \frac{10}{2} \right)^{0.4}$   
$$= 567 \text{ K.}$$

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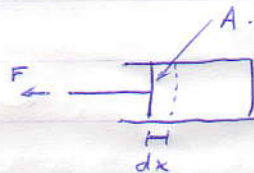
3a. 2 marks The first law states that the change in the internal energy of a system is made up of heat  $Q$  and Work  $W$  flowing into or out of the system to or from the surroundings. Essentially, it is a statement of the conservation of energy and an observation of how the universe works.

$dU$  is the change in the internal energy  $U$  of a system.  $Q$  is the heat entering or leaving the system and is the spontaneous transfer of energy in response to a difference in temperature between the system and its surrounding and involves no net force. Finally  $W$  is the work done by or on the system moving a net force through/over some displacement and also represents an energy flow across the system boundary.

3b. 2 marks the definition of work is the energy required such that a net force  $F$  acts to move its point of application through some displacement  $dx$ .

ie.  $dW = \underline{F} \cdot \underline{dx}$ .

if we take a piston & cylinder



$F = PA$ .  $dV = A dx$

$dW = \underline{F} \cdot \underline{dx} = PA dx$  ( $\underline{F}$  and  $\underline{dx}$  are parallel)  
 $= -PdV$

$dW$  is negative because for an expansion  $dV$  the system uses its internal energy to do work on its surroundings.

the total work  $W = \int dW$   
 $= - \int PdV$ .

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3c.

2 marks

$$\begin{aligned} W &= - \int P dV \\ &= - \int \frac{nRT}{V} dV \quad \text{using the ideal gas law } P = \frac{nRT}{V} \\ &= - nRT \int \frac{dV}{V} \quad \text{since the process is isothermal, then } nRT = \text{constant.} \\ &= - nRT \left[ \ln V \right]_{V_i}^{V_f} \quad \text{To do this question quickly we} \\ & \quad \text{can use some cunning.} \end{aligned}$$

Firstly  $nRT = P_i V_i$  and since  $P_f = \frac{1}{2} P_i$  then  $V_f = 2V_i$  by Boyle's law

$$\begin{aligned} \text{so } W &= - 0.001 \times 1.01 \times 10^5 \left[ \ln 0.002 - \ln 0.001 \right] \\ &= - 70 \text{ J.} \end{aligned}$$

The work done is negative meaning the system uses some of its internal energy to do work on its surroundings to perform this isothermal expansion.

3d. The three heat transport mechanisms are: conduction, convection and radiation

2 marks

**Conduction:** Heat is transferred particle to particle by interparticle collisions and interactions (ie. phonons, lattice vibrations, atom-electron coupling, etc). Typically best in solids and worst in gases, particularly at low pressure. Easily calculated since its proportional to area & temperature difference and inversely proportional to thickness

**convection:** only in fluids not in solids where atoms aren't free to move and not in low pressure gases. Very difficult to treat numerically, but operates by having a packet of fluid which expands, becomes less dense, floats to the top, cools, becomes more dense and then sinks back down creating 'convection cells' which transfer heat. Could be forced or natural, etc.

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3d. Radiation: Is the transfer of heat by the radiation and absorption of photons. All objects radiate photons with an emission given by the blackbody spectrum. As  $T$  increases the total emission increases and the peak wavelength shifts to lower wavelengths, etc.

(\*) Examiner's note: Looking for quality of discussion here so its hard to give a 'fixed' answer for this question.

3e. How much heat do we need to remove from the water

2 marks.

$$\begin{aligned} Q &= MC_w (100^\circ\text{C} - 65^\circ\text{C}) \\ &= 200\text{g} \times 4.18\text{J/g}^\circ\text{C} \times 35^\circ\text{C} \\ &= 29260 \text{ J.} \end{aligned}$$

The heat in taking a mass  $m$  of ice cubes to  $65^\circ\text{C}$  is

$$\begin{aligned} Q &= mL + mC_w (65^\circ\text{C} - 0^\circ\text{C}) \\ &= m(L + C_w \Delta T) \\ m &= \frac{29260}{(333 + 4.18 \times 65)} = \frac{29260}{604.7} \\ &= 48.4\text{g.} \end{aligned}$$

Which means  $48 \times 1\text{g}$  ice cubes.