

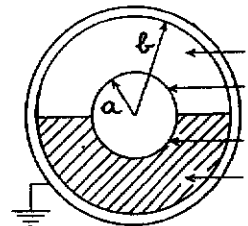
Dielectrics and Capacitors

1. Show that the capacitance per unit length of two coaxial metal cylinders, inner radius a , outer radius b , is $C = 2\pi\epsilon_0/\ln(b/a)$.
2. An air dielectric parallel plate capacitor is charged to 200V. If the area of each plate is 5 cm^2 and the spacing between plates is 0.1 mm, find (a) the charge density σ on each plate; (b) the force pulling the plates together and (c) the total energy stored in the capacitor. (Neglect fringing effects)

[Ans: a) $1.77 \times 10^{-5}\text{ Cm}^{-2}$, b) $8.85 \times 10^{-3}\text{ N}$, c) $8.85 \times 10^{-7}\text{ J}$]

3. A capacitor of two large parallel plates has an internal separation d between the plates. A slab of linear dielectric of dielectric constant κ and thickness $a < d$ is placed on the lower plate. Neglect edge effects. If the surface charge density on the plates is σ show that the potential difference between the plates is $(\sigma/\epsilon_0)[(a/\kappa)+d-a]$.

4. A pair of concentric conducting spheres of radii a and b ($a < b$) has the space between the spheres half-filled with a liquid dielectric of dielectric constant $\kappa = 3.0$, as shown in the diagram. The space above the dielectric may be assumed to have a dielectric constant of 1.0. A total charge $+Q$ is placed on the inner sphere and the outer one is earthed.



- (a) Derive expressions in terms of Q for \mathbf{E} , \mathbf{D} , and \mathbf{P} in both halves of the system.
- (b) In terms of Q what are the surface charge densities σ_{1f} and σ_{2f} of free charge on the inner conductor? (c) Find the potential difference between the spheres.

[Ans: a) $E_1 = E_2 = Q/8\pi\epsilon_0 r^2$, $D_1 = 3Q/8\pi r^2$, $D_2 = Q/8\pi r^2$, $P_1 = Q/4\pi r^2$, $P_2 = 0$,
 (b) $\sigma_{1f} = 3Q/8\pi a^2$, $\sigma_{2f} = Q/8\pi a^2$, (c) $V = (Q/8\pi\epsilon_0)(1/a - 1/b)$]

5. A capacitor with capacitance C_0 is placed in a tank and the following sequence of events occurs:
 - (a) a battery of voltage V is connected to the capacitor;
 - (b) the tank is filled with oil of dielectric constant κ
 - (c) the battery is disconnected and the oil is removed.

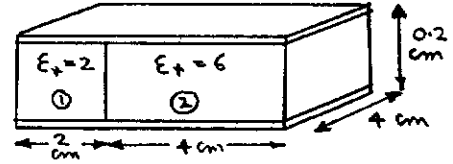
What is the energy (in terms of C_0 and V) stored in the capacitor at each stage? Explain how energy is conserved.

[Ans: (a) $C_0 V^2/2$, (b) $\kappa C_0 V^2/2$, (c) $\kappa^2 C_0 V^2/2$]

6. For a cube of side 2 m with opposite corners at $(1,1,1)$ and $(-1,-1,-1)$ and polarization $\mathbf{P} = x\mathbf{i} + y\mathbf{j}\text{ Cm}^{-2}$ find (a) σ_b on all faces, (b) ρ_b inside the cube, (c) the total bound surface charge and (d) the total bound volume charge.

[Ans: (a) $\sigma_b = 1$ at $x = \pm 1$, $y = \pm 1$, $\sigma_b = 0$ at $z = \pm 1$, (b) $\rho_b = -2$, (c) 16 C, (d) -16C]

7. If 100 V is applied to the parallel-plate capacitor shown in the figure, find (a) E_1 , (b) E_2 , (c) D_1 , (d) D_2 , (e) P_1 , (f) P_2 , (g) C_{total} . (Neglect edge effects]



[Ans: (a) 5×10^4 V/m, (b) 5×10^4 V/m, (c) $10^5 \epsilon_0$ Cm⁻², (d) $3 \times 10^5 \epsilon_0$ Cm⁻², (e) $5 \times 10^4 \epsilon_0$ Cm⁻², (f) $2.5 \times 10^5 \epsilon_0$ Cm⁻², (g) 50 pF]

8. A parallel-plate capacitor, $C = 1 \mu F$, has been charged to 3000 V then disconnected from the charging source. If the dielectric constant κ is 3 by how much will the stored energy be changed by removing the dielectric?

[Ans: Increased by 9 J]

9. A coaxial cable, $r_a = 0.8$ cm, $r_b = 3.0$ cm, has two concentric layers of dielectric between the conductors. The inner layer has $\kappa = 6.0$ for $0.8 \text{ cm} \leq r \leq 1.0$ cm and the outer layer has $\kappa = 3.0$ for $1.0 \text{ cm} \leq r \leq 3.0$ cm. A potential difference of 12.5 kV is applied between the conductors. Find (a) the capacitance per unit length [Hint: treat the cable as two cylindrical capacitors in series], (b) the surface charge density on the inner conductor and (c) the maximum electric field in each dielectric.

[Ans: (a) 138 pF/m, (b) 3.4×10^{-5} Cm⁻², (c) 6.5×10^5 V/m, 1.0×10^6 V/m]