PHYS2050 Problem Sheet 3

The Electric Potential

1. Find expressions for the potential V outside and inside a charged solid sphere of uniform volume charge density ρ_v and radius R.

   [Ans: outside $V(r) = \rho_v R^3/3\epsilon_0 r$; inside $V(r) = \rho_v(3R^2 - r^2)/6\epsilon_0$]

2. An electric field is given by $\mathbf{E} = 2x^2\mathbf{i} + 3y^2\mathbf{j} + 4z\mathbf{k}$. Calculate a) the divergence of $\mathbf{E}$, b) the charge density which gives rise to this field at the points (0,0,0) and (1,1,1); c) the curl of $\mathbf{E}$; and d) the potential difference V(1,1,1)-V(0,0,0).

   [Ans: a) $4x + 6y + 4$, b) $4\epsilon_0, 14\epsilon_0$, c) 0, d) $-11/3$]

3. Find the potential along the axis of a disc of charge density σ and radius b, and from this potential function find the electric field on the axis.

   [Ans: $V = (\sigma/2\epsilon_0)((z^2 + b^2)^{1/2} - z), \mathbf{E} = (\sigma/2\epsilon_0)(1 - z(z^2 + b^2)^{-1/2})\mathbf{k}$]

4. The electric field between two coaxial cylinders is 500 V/m at the inside surface of the outer conductor. Find the potential difference between the conductors if the radii are 2 cm and 5 cm. [Ans: 23 V]

5. Three charges, each of +q, are placed at three corners of a square of side d. a) How much work does it take to bring another charge -3q from far away and place it at the fourth corner? b) How much work is required to assemble the whole configuration of four charges?

   [Ans: a) $-3q^2(2 + 1/\sqrt{2})/4\pi\epsilon_0 d$, b) $-q^2(4 + \sqrt{2})/4\pi\epsilon_0 d$]

6. Find the electrostatic energy of a uniformly charged solid sphere of radius R and total charge Q using two different methods. [Ans: $3Q^2/20\pi\epsilon_0 R$]

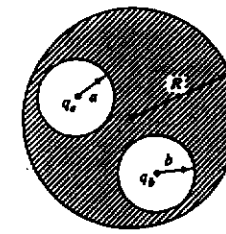
7. Does the superposition principle extend to electrostatic energy? That is, can you add the energies of two charge distributions to get the total energy? [Ans: No]

Conductors

8. An insulated metal sphere 20mm in radius carries a positive charge of 1 nC. It is concentric with an insulated hollow metal sphere having an inner radius of 40 mm and an outer radius of 60 mm. The hollow sphere carries a total negative charge of 0.5 nC. Find a) the charges on the inner and outer surfaces of the hollow sphere, b) the potential difference between the two spheres, and c) the field intensity at distances of 30, 50 and 90 mm from the centre of the spheres.

   [Ans: a) inner: -1 nC, outer: 0.5 nC; b) 225 V; c) $\mathbf{E}_{30} = 10^4\hat{\mathbf{r}}$ V/m: ; $\mathbf{E}_{50} = 0$; $\mathbf{E}_{70} = 550\hat{\mathbf{r}}$ V/m]

9. Two spherical cavities of radii a and b are hollowed out from the interior of a neutral conducting sphere of radius R. Point charges q_a and q_b are placed at the centres of the two cavities as shown. a) Find the surface charges σ_a, σ_b and σ_R on the inner surfaces of the two cavities and the outer surface of the conductor.



b) What is the field within each cavity?

c) What is the force on q_a and q_b?

d) Would any of your answers change if a third charge, q_c, were brought near the conductor? Why?

[Ans: a) $\sigma_a = -q_a/4\pi a^2$, $\sigma_b = -q_b/4\pi b^2$, $\sigma_R = (q_a + q_b)/4\pi R^2$, b) $\mathbf{E}_a = (q_a/4\pi\epsilon_0 r_a^2)\hat{\mathbf{r}}$, $\mathbf{E}_b = (q_b/4\pi\epsilon_0 r_b^2)\hat{\mathbf{r}}$, c) zero, d) σ_R no longer uniform]