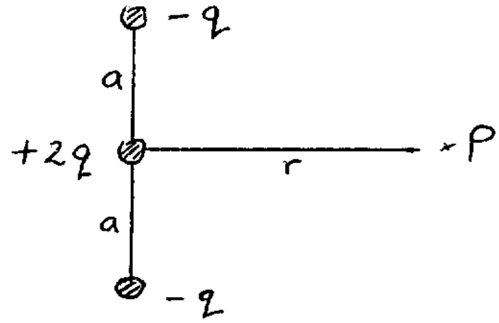
The Electric Field



1. Calculate the electric field at the point P for the arrangement of charges shown in the diagram. Simplify your answer for the case $r \gg a$.

   [Ans: $(q/2\pi\epsilon_0 r^2)(1-(1+(a/r)^2)^{-3/2})$, $3a^2q/4\pi\epsilon_0 r^4$]

2. A line of charge has a charge density λ and is 2L long. Consider the electric field at distance x from the centre of and at right angles to the line of charge. At what distance (in terms of L) from the above line of charge will the field be the same to within a) 10% and b) 1% as the field of an infinite line of charge density λ at the same distance?

   [Ans: a) $x = 0.48L$; b) $x = 0.14L$]

3. Find the electric field at (0,0,2) due to two infinite line charges, both parallel to the x-axis and lying in the $z = 0$ plane. One has $\lambda = 10^{-5}Cm^{-1}$ and is located at $y = 2$ m; the other has $\lambda = -10^{-5}Cm^{-1}$ and is located at $y = -2$ m.

   [Ans: -9×10^4j V/m]

4. A semi-infinite sheet of charge density σ is described by $-\infty < x < 0, -\infty < y < \infty$ in the $z = 0$ plane. Calculate the component of electric field normal to the sheet at a distance z directly above the edge at $x = 0$. Compare your result with the field of a complete plane sheet.

   (Hint: divide the sheet into thin strips and use the formula for a line of charge.

   $$\int z/(x^2+z^2)dx = \tan^{-1}(x/z)\ )$$

   [Ans:$\sigma/4\epsilon_0$]

5. Find **E** on the axis at one end of a uniform cylindrical shell of surface charge density σ. Let the cylinder be of length L and radius R.

   (Hint: Divide the shell into thin rings and use the result for a ring of charge)

   [Ans:$(\sigma/2\epsilon_0)(1-R/(L^2+R^2)^{1/2})$]

6. Use the expressions for the electric fields of an infinite plane sheet and a disc to find the electric field along the z axis of an infinite sheet of uniform charge density σ lying in the $z = 0$ plane, with a circular hole of radius a centred at the origin cut from it.

   [Ans: $\sigma z/2\epsilon_0(z^2+a^2)^{1/2}\mathbf{k}$]

7. Two infinite parallel planes separated by a finite distance carry equal and opposite uniform charge densities $\pm\sigma$. Find the electric field (a) in the region between the planes, and (b) in the region outside the planes.

   (Hint: Superimpose the fields of the individual sheets.)

   [Ans: a) σ/ϵ_0, b) 0]

8. A nonconducting sphere of radius R carries a spherically symmetric charge distribution with density $\rho = \rho_0[1 - (r/R)^2]$. Calculate a) the total charge on the sphere, b) the electric field strength E(r) at a point outside the sphere and c) the electric field E(r) inside the sphere. Sketch E(r) as a function of r.

   [Ans: a) $8\pi\rho_0 R^3/15$, *b)* $(2\rho_0 R^3/15\epsilon_0 r^2)\hat{\mathbf{r}}$, *c)* $(\rho_0/\epsilon_0)(r/3 - r^3/5R^2)\hat{\mathbf{r}}$]

9. A long cylinder carries a charge density which is proportional to the distance r from the axis: $\rho = Ar$ where A is a constant. Find the electric field inside the cylinder as a function of r.

   [Ans: $(Ar^2/3\epsilon_0)\hat{\mathbf{r}}$]

10. Three concentric spherical shells have radii 1 m, 2 m, and 3 m and carry uniformly distributed charges 3 nC, -1 nC and -2 nC respectively. Find **E** in all regions and make a plot of E versus r.

    [Ans: 0, $27/r^2\ V/m$, $18/r^2\ V/m$, 0]