

1. Use the dot product to show that $\mathbf{A} = 2\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$ are perpendicular.
2. Find the unit vector in the direction of $\mathbf{A} \times \mathbf{B}$ when $\mathbf{A} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{B} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

[Ans: $(-\mathbf{i} + 2\mathbf{j} - \mathbf{k})/\sqrt{6}$]

3. Given the vectors \mathbf{A} , \mathbf{B} and \mathbf{C} , where $\mathbf{A} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{B} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{C} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, find $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ and $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$. Are they the same?

[Ans: Yes, 0]

Theorems of Differential Calculus

4. Using Cartesian coordinates, show that $\nabla \cdot (f\mathbf{a}) = (\nabla f) \cdot \mathbf{a} + f(\nabla \cdot \mathbf{a})$
5. Using Cartesian coordinates, show that $\nabla \times (\nabla f) = 0$

Theorems of Integral Calculus

6. Check the gradient theorem for the scalar field $T = x^2 + 4xy + 2yz^3$ and the paths
 - a) $(0, 0, 0) \rightarrow (1, 0, 0) \rightarrow (1, 1, 0) \rightarrow (1, 1, 1)$, and
 - b) $(0, 0, 0) \rightarrow (0, 0, 1) \rightarrow (0, 1, 1) \rightarrow (1, 1, 1)$

[Ans: 7]

7. Check the Divergence theorem for the vector field $\mathbf{A} = xyz\mathbf{i} + 2yz\mathbf{j} + 3zx\mathbf{k}$ and a cubic volume of side 2 units with opposite corners at $(0, 0, 0)$ and $(2, 2, 2)$.

[Ans: 48]

8. Check Stokes' Theorem for the vector field $\mathbf{A} = (2xz + 3y^2)\mathbf{j} + 4yz^2\mathbf{k}$ and a square of side 1 unit with opposite corners at $(0, 0, 0)$ and $(0, 1, 1)$.

[Ans: $\pm 4/3$]

Surface and Volume Integrals

9. Find the total charge on a sphere of radius R if the volume charge density inside it is given by $\rho_v = 4Ar \text{ cm}^{-3}$ where r is the distance in metres from the centre of the sphere. What are the units of A ?
10. Find the total charge on a cylinder of radius R and length L if the volume charge density within it is $A/r \text{ cm}^{-3}$, where r is the distance from the axis of the cylinder, and the cylinder has surface charge density of $Br^2 \text{ cm}^{-2}$ on each flat end. What are the units of A and B ?

[Ans: $2\pi ALR + \pi BR^4 \text{ C}; \text{Cm}^{-2}; \text{Cm}^{-4}$]