PHYS2050 Electromagnetism

Part 2. Problem Sheet

1. a) Find the magnetic field at the centre of a square loop of side R, which carries a steady current, I.

   b) Find the magnetic field at the centre of a regular n-sided polygon carrying a steady current, I. Let R be the distance from the centre, to the centre of any side. Consider the limit as $n \to \infty$.

2. A steady current I flows down a long cylindrical wire of radius R. Find the magnetic field, both inside and outside the wire, if

   a) the current is uniformly distributed across the wire.

   b) the current is uniformly distributed over the outside surface of the wire.

   c) the current is distributed in such a way that J is proportional to r, the distance from the axis.

3. A thick slab extending from $z = -a$ to $z = a$, and infinite in the xy plane, carries a uniform volume current $\mathbf{J} = J\mathbf{i}$. Find the magnetic field both inside and outside the slab.

4. (See G. problem 5.46) A Helmholtz coil consists of two co-axial circular loops of radius R, each carrying a current I, and separated by a distance d. Find the magnetic field as a function of distance along their common axis, near the mid-point (taken as $z = 0$). Show that the first derivative of B with respect to z is zero at the mid point. Then show that if $d = R$, then the second derivative of B with respect to z is also zero.

5. What current density would produce a constant azimuthal potential, $\mathbf{A} = k\,\hat{\phi}$ (in cylindrical coordinates)?

6. If $\mathbf{B}$ is uniform, show that $\mathbf{A} = -1/2\,(\mathbf{r}\times\mathbf{B})$, where $\mathbf{r}$ is a vector from the origin to the point in question. That is, show that $\nabla.\mathbf{A} = 0$, and $\nabla\times\mathbf{A} = \mathbf{B}$.

7. A wire of radius R carries a current I, uniformly distributed across it. Find the vector potential <u>inside</u> it. (Choose $\mathbf{A} = \mathbf{0}$ at $r = R$.)

8. An infinitely long circular cylinder carries a uniform magnetization, $\mathbf{M}$, parallel to its axis. Calculate the resulting $\mathbf{B}$ field, inside and outside the cylinder.

9. An infinitely long cylinder of radius R carries a frozen in magnetization parallel to the axis defined by $\mathbf{M} = kr\,\hat{z}$, where k is a constant, and r is the distance from the axis. Find the magnetic field inside and outside the cylinder

   a) by locating the bound currents, and then calculating the fields they produce;

   b) using Ampere’s law, in the form (5.7’), to find $\mathbf{H}$, and then find $\mathbf{B}$ from (5.6).

10. An infinitely long cylinder of radius R carries a magnetization defined by $\mathbf{M} = kr^2\,\hat{\phi}$, where k is a constant, r is the distance from the axis, and $\hat{\phi}$ is the circumferential unit vector. Calculate the resulting $\mathbf{B}$ field, inside and outside the cylinder.

11. A coaxial cable consists of two very long concentric hollow cylinders, of radii *a* and *b* ($a<b$), separated by linear insulating material of magnetic susceptibility χ_m. A current *I* flows to the right along the inner cylinder, and returns along the outer one. (In each case the current is uniformly distributed.) Find the magnetic field in the region between the tubes. As a check, calculate the magnetization and bound currents, and confirm that, along with the free currents, they generate the correct field.

12. A current *I* flows down a long straight wire of radius *R*. If the wire is made of material with susceptibility χ_m, and the current is distributed uniformly, what is the magnetic field a distance *r* from the centre? Find all the bound currents. What is the net bound current flowing down the wire?

13. A square loop of wire, of side length *s*, lies on a table near a very long straight wire which carries a current *I*. The nearest side is parallel to the wire, and a distance *d* from it. Find the magnetic flux through the loop. If the loop is now moved directly away from the wire, at a speed *v*, what EMF (magnitude and direction) is generated?

14. A circular loop of wire, of radius *R*, lies in the *xy* plane, with its centre at the origin. In this region there is a magnetic field given by $\mathbf{B}(r,t) = c\, r\, t^3\, \mathbf{k}$ (where *c* is a constant and **k** is the unit vector). Find the flux through the loop. Find the emf induced in the loop.

15. A long solenoid of radius *a*, carrying *N* turns per unit length, has a wire of total resistance *R* looped around it. The current in the solenoid is decreasing at a constant rate – i.e. $dI/dt = k$ (constant). What current flows in the loop?

16. (See G. problem 7.53) A simple transformer consists of two coils wrapped around a cylinder so that any flux through one also penetrates the other. If the current *I* in the primary coil (which has N_1 turns) is changing, show that the emf in the secondary (which has N_2 turns) is given by $\varepsilon_2 / \varepsilon_1 = N_2 / N_1$.

17. Find the self-inductance per unit length of a long solenoid of radius *R*, carrying *n* turns per unit length.

18. Find the energy stored in a length *l* of the solenoid of the previous problem, when a current *I* flows: (a) using eq.(6.12); (b) using eq.(6.13); (c) using eq.(6.15).

19. A long cable carries a current *I* in one direction, uniformly distributed over its (circular) cross section. This current returns along the surface (separated by an insulating skin). Find the self-inductance per unit length.

20. A capacitor made from parallel circular plates of radius *a* and separation *s* ($s<<a$), is inserted in a long straight wire carrying a constant current *I*. As the capacitor charges up, find the induced magnetic field midway between the plates, as a function of distance from the centre, both ‘inside’ and ‘outside’ (i.e $r<a$ and $r>a$).