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Electromagnetism PHYS2050

3 Electric Field in Matter

3.1 Electric Polarization

- **Conductors** The charge carriers (electrons or holes) can move freely. In case of an external electric field, the charges try to compensate the external field by building up induced surface charges.
- **Insulators/Dielectrics** All charge carriers are attached to the atoms (molecules). They can still move a bit inside an atom (molecule). Through this displacement, microscopic dipole moments are induced. The atoms get polarized.

Dielectrics

1. Induced atomic dipole moments

Atoms consist of a positively charged atomic core (neutrons and positively charged protons) and a negatively charged electronic cloud, which surrounds the atomic core.

In an external electric field the positively charged core is pushed slightly with the direction of the external field while the electronic cloud is moved against the direction of the external field. Due to this charge displacements the atoms become polarized, forming atomic dipoles.

This atomic polarization can be described trough an atomic dipole moment. Note that the atomic dipole moment is parallel to the external electric field.

$$\vec{p} \;=\; \alpha\vec{E}$$

α: atomic polarizability

Some experimentally determined atomic polarizabilities are:

H	0.667	Ne	0.396
He	0.205	Na	24.1
Li	24.3	Ar	1.64
Be	5.60	K	43.4
C	1.76	Cs	59.6

2. Alignment of Polar Molecues

Some molecules have a built in electric dipole moment. Typical examples are organic molecules.

The most prominent example is water H_2O, where the $H - O - H$ atoms form an angle of 105°. The dipole moment of water is unusually large: $6.1 \cdot 10^{-30}\ Cm$.

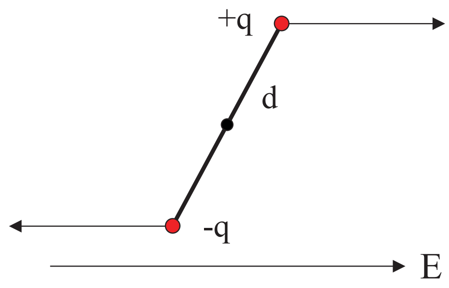

Figure 3.1: Torque caused by an external electric field on an electric dipole.

The torque, which is caused by an external electric field is:

$$\begin{aligned} \vec{N} &= \left(\vec{r}_+ + \vec{F}_+\right) + \left(\vec{r}_- + \vec{F}_-\right) \\ &= \left[(\vec{d}/2) \times (q \cdot \vec{E})\right] + \left[(-\vec{d}/2) \times (-q \cdot \vec{E})\right] \\ &= q \cdot \vec{d} \times \vec{E} = \vec{p} \times \vec{E} \end{aligned}$$

3.2 Electric field of an electrically polarized object

How large is the electric field created by a polarized object?

Potential of a single electric dipole:

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{e}_r \cdot \vec{P}}{|\vec{r}|^2}$$

Now let us assume that we have a dipole moment $\vec{p} = \vec{P}\ d\tau'$ in each volume element $d\tau'$, i.e. the volume of one single atom or molecule:

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\vec{e}_r \cdot \vec{P}(\tau')}{|\vec{r} - \vec{\tau}'|^2}\ d\vec{\tau}'$$

In a previous chapter we have demonstrated already that:

$$\nabla' \left(\frac{1}{|\vec{r} - \vec{r}'|}\right) = + \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^2}$$

Note that ∇' is the derivative of the function by $\vec{r}'$. This explains the positive sign.

Using this expression, the electric potential can be written as:

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int\limits_V \vec{P}(\vec{\tau}') \, \nabla' \frac{1}{|\vec{r} - \vec{\tau}'|} \, d\vec{\tau}'$$

This gives finally:

$$\begin{aligned} \phi(\vec{r}) &= \frac{1}{4\pi\varepsilon_0} \left[\int\limits_V \nabla' \frac{\vec{P}(\vec{\tau}')}{|\vec{r} - \vec{\tau}'|} \, d\vec{\tau}' - \int\limits_V \frac{1}{|\vec{r} - \vec{\tau}'|} \left(\nabla' \vec{P}(\vec{\tau}') \right) \, d\vec{\tau}' \right] \\ &= \underbrace{\frac{1}{4\pi\varepsilon_0} \oint\limits_S \frac{\vec{P}(\vec{\tau}')}{|\vec{r} - \vec{\tau}'|} \, d\vec{a}'}_{\text{Surface Potential}} - \underbrace{\frac{1}{4\pi\varepsilon_0} \int\limits_V \frac{1}{|\vec{r} - \vec{\tau}'|} \left(\nabla' \vec{P}(\vec{\tau}') \right) \, d\vec{\tau}'}_{\text{Volume Potential}} \end{aligned}$$

The first term looks like the potential of a surface charge:

$$\sigma_d = \vec{P} \cdot \vec{n} \quad ,$$

whereas the second term has the form of the potential of a volume charge:

$$\varrho_d = -\nabla \cdot \vec{P} \quad .$$

Note that in both cases we are dealing with a dipole moment instead of a charge.

The potential of a polarized object is the same as that produced by a volume charge density $\varrho_d = -\nabla \cdot \vec{P}$ plus a surface charge density $\sigma_d = \vec{P} \cdot \vec{n}$.

The 'trick' is to determine these 'dipole' charge densities (density of bound and displaced electronic charges).

The electric field caused by a polarized object can then be calculated as if there is a volume+surface charge density, which acts against the external electric field.

Physical interpretation of the bond charges

Volume polarization + surface polarization:

At the ends (surface) there are two charges left, + and -. They cause the surface polarization.

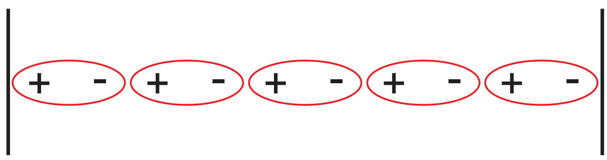

Figure 3.2: Physical interpretation for the volume and surface polarization.

3.3 The Electric Displacement

An external electric field creates bound volume dipole moments and bound surface dipole moments in a dielectric material.

$$\varrho \quad = \quad \varrho_{\text{volume}} + \varrho_{\text{surface}}$$

Gauss Law:

$$\begin{aligned} \varepsilon \, \nabla \cdot \vec{E} &= \varrho = \varrho_v + \varrho_s \;=\; -\nabla \cdot \vec{P} + \varrho_s \\ \varrho_s &= \nabla \cdot \left(\varepsilon_0 \vec{E} + \vec{P} \right) \end{aligned}$$

Dielectric Displacement D:

$$\boxed{\vec{D} \;=\; \varepsilon_0 \vec{E} + \vec{P} \qquad\qquad \nabla \cdot \vec{D} \;=\; \varrho}$$

This is the Gauss law (one of the Maxwell equations) in a dielectric medium.

Problem: Just to replace $\vec{E}$ by $\vec{D}$ is not sufficient :

Coulomb Law:

$$\vec{D}(\vec{r}) \;\neq\; \frac{1}{4\pi\varepsilon} \int_V \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} \, \varrho(\vec{r}') \, d\vec{r}'$$

curl of $\vec{D}$:

$$\nabla \times \vec{D} \;=\; \varepsilon_0 \underbrace{(\nabla \times \vec{E})}_{0} \;+\; \nabla \times \vec{P} \;=\; \nabla \times \vec{P}$$

3.4 Linear Dielectrics

In most materials the polarization is proportional to the electric field:

$$\vec{P} \;=\; \varepsilon_0 \cdot \chi_{el.} \cdot \vec{E}$$

$\chi_{el.}$ is the electric susceptibility of the medium.

The prefactor ε_0 ensures that $\chi_{el.}$ is a dimensionless factor.

Dielectric Displacement:

$$\vec{D} \;=\; \varepsilon_0 \cdot \vec{E} \;+\; \vec{P} \;=\; \varepsilon_0 \cdot \vec{E} \;+\; \varepsilon_0 \cdot \chi_{el.} \cdot \vec{E} \;=\; \varepsilon_0 \;(1 + \chi_{el.})\; \vec{E}$$

$$\boxed{\vec{D} \;=\; \varepsilon \cdot \vec{P} \qquad \text{where :} \qquad \varepsilon \;=\; \varepsilon_0 \;(1 + \chi_{el.})}$$

$\chi_{el.}$ is the **electric susceptibility** of the medium.

ε is the **electric permittivity** of the medium and

$$\varepsilon_r \;=\; 1 + \chi_{el.} \;=\; \frac{\varepsilon}{\varepsilon_0}$$

is the **relative electric permittivity** of the medium or **dielectric constant**.

Most materials are linear dielectrics. Materials with nonlinear dielectric properties are rare cases and offer fascinating possibilities for industrial applications (like nonlinear optical materials).

Since the dielectric displacement $\vec{D}$ and the electric field $\vec{E}$ are three dimensional vectors, the dielectric constant ε is in general a 3×3 tensor, which contains 1,2,3,4, or 6 components, depending on the symmetry of the crystal structure of the material.

Some experimentally determined dielectric constants:

Vacuum	1	Diamond	5.7
Helium	1.000065	Salt (NaCl)	5.9
Neon	1.00013	Silicon	11.8
Nitrogen	1.00055	Water	80.1
Air	1.00054	Ice (-30^0 C)	99

3.5 Energy and Force in Dielectric Materials

Example: dielectric medium in a capacitor:

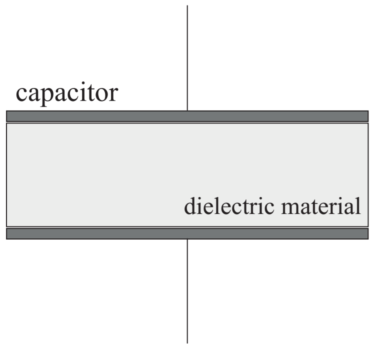

Figure 3.3: Dielectric material inside a capacitor.

The energy of a capacitor which is filled with a dielectric material is:

$$W = \frac{1}{2} C V^2 \qquad C = \varepsilon_r \cdot C_{\text{Vacuum}}$$

Additional charges need to be 'pumped' to the capacitor in order to achieve the same given potential since the induced electric dipole moment of the dielectric medium partly compensates the electric field inside the capacitor. The energy of the capacitor with a dielectric medium between its conducting plates can therefore be written as:

$$W = \frac{\varepsilon}{2} \int (\vec{E}(\vec{r}))^2 \, d\vec{r} = \frac{\varepsilon_0}{2} \int \varepsilon_r (\vec{E}(\vec{r}))^2 \, d\vec{r} = \frac{1}{2} \int \vec{D} \cdot \vec{E} \, d\vec{r}$$

Force on a dielectric medium

A dielectric medium which is partly placed inside a capacitor experiences a force, which pushes it further inside the capacitor.

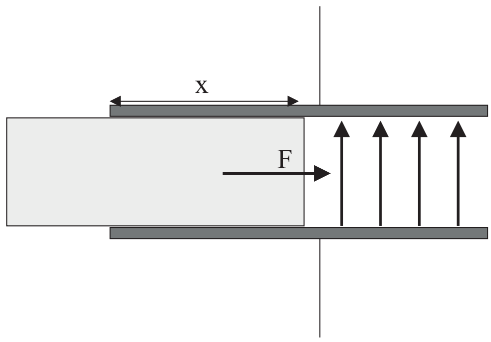

Figure 3.4: Force on a dielectric medium which is partly inside a capacitor.

This force can be calculated by using:

$$F = -\left(\frac{dW}{dx}\right)_{Q=const.}$$