# $\underline{E}$

**#1** Potential V outside and inside a charged solid sphere

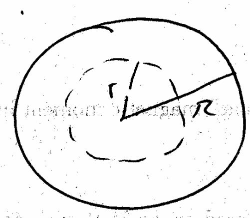
Radius $= R$ , charge density $= \rho$

a) $V_{outside} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_{sphere}}{r}$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{\frac{4}{3}\pi R^3 \rho}{r}$$

$$= \frac{\rho R^3}{3\varepsilon_0 r}$$

b) $V_{inside}$.




$E_{inside} = ?$

$$4\pi r^2 E = \frac{Q_{encl}}{\varepsilon_0}$$

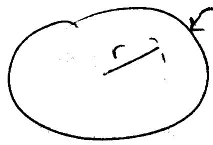
$$E(r) = \frac{Q_{encl}}{4\pi\varepsilon_0 r^2}$$

$$= \tfrac{4}{3}\pi r^3 \rho \,/\, 4\pi\varepsilon_0 r^2$$

$$= \rho r / 3\varepsilon_0$$

## $\underline{E}$

#1




We know $V(r=R)$ from (a)

$$V(R) = \frac{\rho R^3}{3\varepsilon_0 R} = \frac{\rho R^2}{3\varepsilon_0}$$

So

$$V(r) = V(R) - \int_R^r \underline{E} \cdot d\underline{l}$$

$$= \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho}{3\varepsilon_0}\int_R^r r\,dr$$

$$= \frac{\rho R^2}{3\varepsilon_0} - \frac{\rho}{3\varepsilon_0}\left[\frac{1}{2}\left(r^2 - R^2\right)\right]$$

$$= \frac{\rho}{6\varepsilon_0}\left[3R^2 - r^2\right]$$

$\underline{E}$

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$$\underline{E} = 2x^2\,\underline{\hat{i}} + 3y^2\,\underline{\hat{j}} + 4z\,\underline{\hat{k}}$$

a) $\text{div}\ \underline{E} = ?$

$$\underline{\nabla}\cdot\underline{E} = \frac{\partial}{\partial x}(2x^2) + \frac{\partial}{\partial y}(3y^2) + \frac{\partial}{\partial z}(4z)$$

$$= 4x + 6y + 4$$

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b) $\rho$ which gives rise to this $\underline{E}$

at $(0\ 0\ 0)$ and $(1\ 1\ 1)$

$$\underline{\nabla}\cdot\underline{E} = \rho/\varepsilon_0$$

So, at $(0\ 0\ 0)$ $\quad \rho = 4\varepsilon_0$

at $(1\ 1\ 1)$ $\quad \rho = (4+6+4)\varepsilon_0 = 14\varepsilon_0$

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c) $\text{curl}\ \underline{E} = 0$

$\underline{E}$

#2 d) Potential difference

$$V(1,1,1) - V(0,0,0)$$

$$V(b) - V(a) = -\int_a^b \underline{E} \cdot d\underline{l}$$

$$d\underline{l} = \hat{\underline{i}}\, dx + \hat{\underline{j}}\, dy + \hat{\underline{k}}\, dz$$

$$\therefore \quad \Delta V = -\left[ \int_0^1 2x^2 dx + \int_0^1 3y^2 dy + \int_0^1 4z\, dz \right]$$

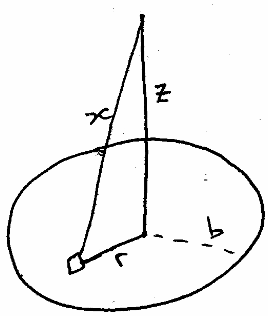
$$= -\left[ \tfrac{2}{3} x^3 \Big|_0^1 + y^3 \Big|_0^1 + 2z^2 \Big|_0^1 \right]$$

$$= -3\tfrac{2}{3}$$

# $\underline{E}$

33 V along axis of a disc of charge density $\rho$ and radius b

($\rho \to \sigma$)




$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{x} \, da$$

$$x = \sqrt{r^2 + z^2}$$

$$da = r \, dr \, d\phi$$

$$V = \frac{1}{4\pi\varepsilon_0} \int_0^{2\pi} \int_0^{b} \frac{\rho r}{\sqrt{r^2+z^2}} \, dr \, d\phi$$

$$= \frac{\rho}{2\varepsilon_0} \int_0^{b} \frac{r \, dr}{\sqrt{r^2+z^2}}$$

$$= \frac{\rho}{2\varepsilon_0} \left[ \sqrt{r^2+z^2} \right]_0^b$$

$$= \frac{\rho}{2\varepsilon_0} \left[ \sqrt{b^2+z^2} - z \right]$$

3 (b) $\underline{E}$ along the axis.

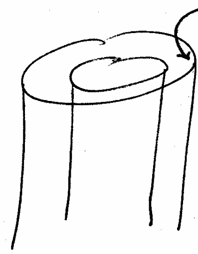
$$\underline{E} = -\underline{\nabla} V$$

$$= \frac{-\rho}{2\varepsilon_0} \frac{d}{dz}\left[\sqrt{b^2+z^2} - z\right]$$

$$= \frac{-\rho}{2\varepsilon_0}\left[\frac{z}{\sqrt{b^2+z^2}} - 1\right]$$

$$= \frac{\rho}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{b^2+z^2}}\right] \hat{\underline{k}}$$

24 2 coaxial cylinders



$E = 500\ \mathrm{Vm^{-1}}$ at the inside surface of the outer conductor

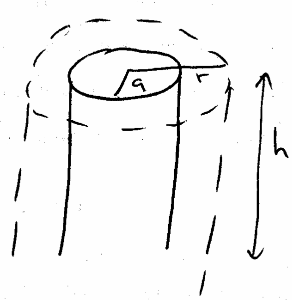
radii = 2 cm + 5 cm

Potential difference between conductors = ?

Need $E(r)$ between cylinders

$$\underset{\sim}{E} = E(r)\,\hat{\underline{r}}$$

$\therefore$ cylindrical Gaussian surface inside cylinder gap




end-caps = 0

$\therefore$ central bit

$$\int \underline{E} \cdot d\underline{a} = \frac{Q_{encl}}{\varepsilon_0}$$

assume $\lambda$ $\mathrm{Cm^{-1}}$

$$\therefore 2\pi r h E = \frac{\lambda h}{\varepsilon_0}$$

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$\underline{E}$

Q4/ We know that $E = 500\ Vm^{-1}$
when $r = 5$ cm

$$\therefore \quad \lambda = 2\pi \varepsilon_0 r E$$
$$= 2\pi\varepsilon_0 \times 0.05 \times 500 = 50\pi\varepsilon_0 \quad Cm^{-1}$$

$$\Delta V = V(5) - V(2) = -\int_2^5 \underline{E} \cdot d\underline{r}$$

$$= -\frac{\lambda}{2\pi\varepsilon_0} \int_2^5 \frac{dr}{r}$$

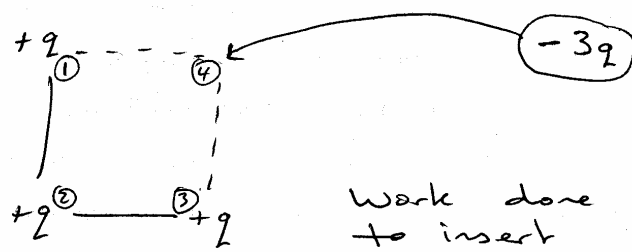
$$= -\frac{50\pi\varepsilon_0}{2\pi\varepsilon_0} \left[ \ln r \right]_{0.02}^{0.05}$$

(r in metres)

$$|\Delta V| = 22.91\ V$$

## E

#5 square. edge = d




Work done = ?
to insert
−3q

V at the 4th corner

$$= \sum_{i=1}^{3} V_i$$

$$= \frac{+q}{4\pi\varepsilon_0}\left[\frac{1}{d} + \frac{1}{d\sqrt{2}} + \frac{1}{d}\right]$$

$$\therefore\ U = QV = -3q \cdot \frac{q}{4\pi\varepsilon_0 d}\left[2 + \frac{1}{\sqrt{2}}\right]$$

$$= \frac{-3q^2}{4\pi\varepsilon_0 d}\left[2 + \frac{1}{\sqrt{2}}\right]$$

E

#5 (b) Work to assemble the 4 charges

$$W = \frac{1}{2} \sum_{i} \sum_{j \neq i} \frac{q_i q_j}{r_{ij}} \cdot \frac{1}{4\pi\varepsilon_0}$$

$$= \frac{1}{8\pi\varepsilon_0} \left\{ \left[ \frac{q^2}{d} + \frac{q^2}{d\sqrt{2}} - \frac{3q^2}{d} \right]_{i=1} \right.$$

$$+ \left[ \frac{q^2}{d} + \frac{q^2}{d} - \frac{3q^2}{d\sqrt{2}} \right]_{i=2}$$

$$+ \left[ \frac{q^2}{d\sqrt{2}} + \frac{q^2}{d} - \frac{3q^2}{d} \right]_{i=3}$$

$$\left. + \left[ -\frac{3q^2}{d} - \frac{3q^2}{d\sqrt{2}} - \frac{3q^2}{d} \right]_{i=4} \right\}$$

$$= \frac{q^2}{8\pi\varepsilon_0 d} \left[ -8 - \frac{4}{\sqrt{2}} \right]$$

$$= \frac{-q^2}{4\pi\varepsilon_0 d} \left[ 4 + \sqrt{2} \right]$$

# $\underline{E}$

**#6** Electrostatic energy of a uniformly charged solid sphere of radius $R$ and total charge $Q$

$$W = \frac{\varepsilon_0}{2} \int E^2 \, d\tau$$

$$E(r) \text{ outside sphere} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E(r) \text{ inside sphere} = ?$$

$$\int \underline{E} \cdot \underline{da} = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \cdot \frac{r^3}{R^3}$$

$$E(r) = \frac{Q r^3}{4\pi\varepsilon_0 r^2 R^3}$$

$$= \frac{Q r}{4\pi\varepsilon_0 R^3}$$

$$\therefore W = \frac{\varepsilon_0}{2} \left[ \int_{inside} E^2 d\tau + \int_{outside} E^2 d\tau \right]$$

$$d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$\underline{E}$

#6

$$W = \frac{\varepsilon_0}{2}\left\{\left[\frac{Q}{4\pi\varepsilon_0 R^3}\right]^2 \cdot 2\int r^4\,dr\right.$$

from $\int_0^{\pi} \sin\theta\,d\theta$

$\int d\phi$

$$\left. + \left[\frac{Q}{4\pi\varepsilon_0}\right]^2 \cdot 2\int \frac{dr}{r^2}\right\} \cdot 2\pi$$

$$= \frac{Q^2 \cdot \varepsilon_0}{(4\pi\varepsilon_0)^2}\left[\frac{1}{R^6}\int_0^R r^4\,dr + \int_0^R \frac{dr}{r^2}\right] \cdot 2\pi$$

$$= \frac{Q^2}{16\pi^2\varepsilon_0}\left[\frac{R^5}{5R^6} + \frac{1}{R}\right] \cdot 2\pi$$

$$= \frac{Q^2}{8\pi\varepsilon_0} \cdot \frac{6}{5R} = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

$\underset{\sim}{E}$

#7 Superposition Principle for Energy?

$$U = \frac{\varepsilon_0}{2} \int E^2 \, d\tau$$

if $\underset{\sim}{E} = \underset{\sim}{E}_1 + \underset{\sim}{E}_2$

$$E^2 = (\underset{\sim}{E}_1 + \underset{\sim}{E}_2) \cdot (\underset{\sim}{E}_1 + \underset{\sim}{E}_2)$$

$$= E_1^2 + E_2^2 + 2 \underset{\sim}{E}_1 \cdot \underset{\sim}{E}_2$$

$$\therefore \quad U = U_1 + U_2 + \underbrace{\varepsilon_0 \int \underset{\sim}{E}_1 \cdot \underset{\sim}{E}_2 \, d\tau}_{\text{extra term}}$$

$\therefore$ No Superposition Principle for Energy

8/ Insulated metal sphere $r = 20\,mm$ carries $q = +1\,nC$. Concentric with an insulated hollow metal sphere $r_{inner} = 40\,mm$ & $r_{outer} = 60\,mm$

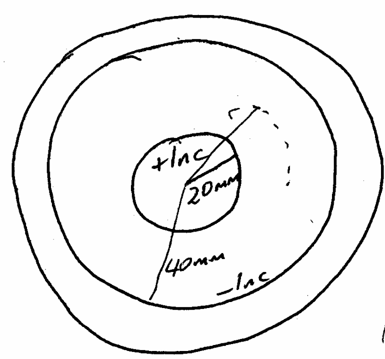
$q_{\substack{hollow \\ sphere}} = -0.5\,nC$

a). charges on inner and outer surfaces of hollow sphere

$q_{inner} = -1\,nC$ to "balance" central sphere

$\therefore q_{outer} = +0.5\,nC \quad \because q_{total} = -0.5\,nC$

b) P.D. between the spheres




$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$= \frac{9\times10^9 \times 1\times10^{-9}}{r^2} = \frac{9}{r^2}$$

$$|\Delta V| = \left| -\int_{20mm}^{40mm} \frac{9}{r^2}\,dr \right|$$

8/

$$|\Delta V| = \left| -\int_{20mm}^{40mm} \frac{9\,dr}{r^2} \right|$$

$$= 9 \left| \frac{1}{r} \right|_{20mm}^{40mm}$$

$$= 225\ V$$

c) $E$ at 30mm, 50 mm - 90mm

$$E \text{ at } 30\text{ mm} = \frac{9}{(0.03)^2} = 10^4\ Vm^{-1}$$

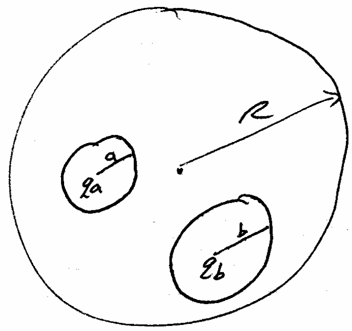
$E$ at 50mm $= 0$ $\because$ inside the conductor and $Q_{encl} = 0$

$E$ at 90mm

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{9\times10^9 \times 0.5\times10^{-9}}{0.09^2}$$

$$= 555\ Vm^{-1}$$

9/




charges $q_a$ and $q_b$ induce corresponding charge densities on the inner surfaces of the cavities ($\rho_{sa} = \rho_a$ etc).

a) $\rho_a = ?$ charge $= -q_a$

surface area $= 4\pi a^2$

$$\rho_a = \frac{-q_a}{4\pi a^2}$$

$$\rho_b = \frac{-q_b}{4\pi b^2}$$

$\rho_{sR}$ = outer surface of conducting sphere

$\therefore$ neutrality $\rightarrow$ $\rho_{sR} = \dfrac{Q}{4\pi R^2} = \dfrac{(q_a + q_b)}{4\pi R^2}$

b/ $E$ inside cavities

$$E_a = \frac{q_a}{4\pi\varepsilon_0 r^2}$$


$$E_b = \frac{q_b}{4\pi\varepsilon_0 r^2}$$

c/ forces on $q_a$ and $q_b$ $= 0$

$\therefore$ inside sphere and $\rho_{sa}$

$\rightarrow$ net force $= 0$

d/ bring $q_c$ near the conductor

$\rho_{sa}$ no longer uniform