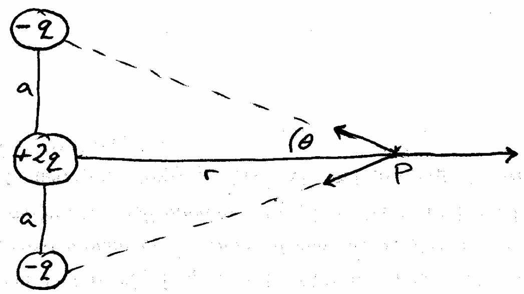
Electric Field

1/



only horizontal component.

$$E(+2q) = \frac{1}{4\pi\varepsilon_0} \frac{2q}{r^2} \quad \rightarrow$$

$$E(-q) = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r^2+a^2)} \quad \nwarrow \quad \text{and} \quad \swarrow$$

$$\cos\theta = \frac{r}{(r^2+a^2)^{1/2}}$$

$$\therefore E_{net} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{r^2} - \frac{2qr}{(r^2+a^2)^{3/2}}\right]$$

$$= \frac{q}{2\pi\varepsilon_0}\left[\frac{1}{r^2} - \underbrace{\frac{r}{(r^2+a^2)^{3/2}}}_{\frac{1}{r^2\left(1+\frac{a^2}{r^2}\right)^{3/2}}}\right]$$

$$\therefore E_{net} = \frac{q}{2\pi\varepsilon_0 r^2}\left[1 - \left(1+\frac{a^2}{r^2}\right)^{-3/2}\right]$$

1/ Case $r \gg a$

$$\left(1 + \frac{a^2}{r^2}\right)^{-3/2} \sim 1 - \frac{3}{2}\frac{a^2}{r^2}$$

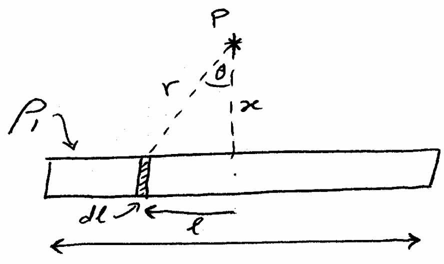
{Binomial $(1+x)^n \sim 1 + nx$}

$$\therefore E = \frac{q}{2\pi \varepsilon_0 r^2}\left[1 - \left(1 - \frac{3}{2}\frac{a^2}{r^2}\right)\right]$$

$$= \frac{3 q a^2}{4\pi \varepsilon_0 r^4}$$

$\underset{\sim}{E}$ field

2/ Line of charge



Compare with ∞ line of charge

only vertical components $\because$ symmetry

$$dE = \frac{dq}{4\pi\varepsilon_0 r^2} \cos\theta$$

$$= \frac{\rho\, dl}{4\pi\varepsilon_0 r^2} \cdot \frac{x}{r}$$

$$= \frac{\rho x}{4\pi\varepsilon_0} \frac{dl}{r^3} \qquad r = (x^2+l^2)^{1/2}$$

$$= \frac{\rho x}{4\pi\varepsilon_0} \frac{dl}{(x^2+l^2)^{3/2}}$$

$$E = \frac{\rho x}{4\pi\varepsilon_0} \int_{-L}^{+L} \frac{dl}{(x^2+l^2)^{3/2}}$$

$$= \frac{\rho x}{4\pi\varepsilon_0} \left[\frac{l}{x^2\sqrt{x^2+l^2}} \right]_{-L}^{+L} = \frac{\rho \cancel{x}}{4\pi\varepsilon_0 x} \left(\frac{2L}{\sqrt{x^2+L^2}} \right)$$

$\underline{E}$ field

2/

want x s.t. $E = 0.9\ E_\infty$ (i.e. within 10%)

$$E_\infty = \frac{\rho}{2\pi\varepsilon_0 x} \quad \text{(lectures)}$$

$$\frac{\rho L}{2\pi\varepsilon_0 x\sqrt{x^2+L^2}} = 0.9\ \frac{\rho}{2\pi\varepsilon_0 x}$$

$$L = 0.9\sqrt{x^2+L^2}$$

$$\Rightarrow \quad x = 0.484\ L$$

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within 1%

$$E = 0.99\ E_\infty$$

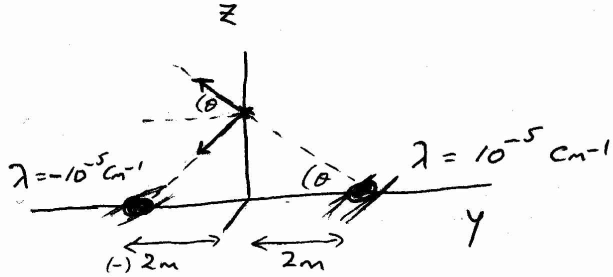
$$L = 0.99\sqrt{x^2+L^2}$$

$$\Rightarrow \quad x = 0.1425\ L$$

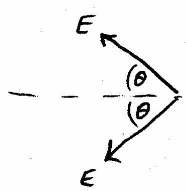
$\underline{E}$ field

3/ $P = (0\ 0\ 2)$

2 ∞ line charges, // x, z = 0 plane



$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad r = \sqrt{2^2+2^2} = \sqrt{8}$$



$$\cos\theta = \frac{2}{\sqrt{8}}$$

$$E_{net} = 2 \cdot \frac{\lambda}{2\pi\varepsilon_0\sqrt{8}} \cdot \frac{2}{\sqrt{8}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} = 10^{-5} \times 9 \times 10^{9}$$

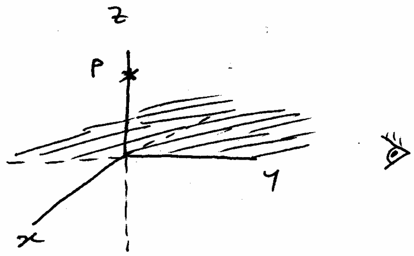
$$= 9 \times 10^{+4}\ \text{Vm}^{-1}$$

$-y$ dir^n

$\underline{E}$ field

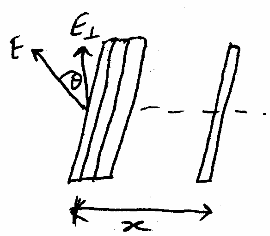
4/ semi-∞ sheet of charge

$$-\infty < y < +\infty$$
$$-\infty < x < 0$$
$$z = 0$$



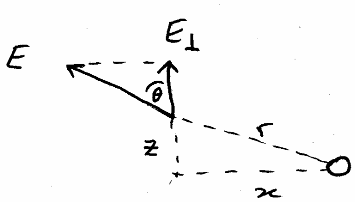
Calculate E_{normal} to sheet at height z - from edge.

divide sheet into thin strips. View from side



each strip creates an E field $\dfrac{\lambda}{2\pi\varepsilon_0 r}$

$$r = \sqrt{x^2 + z^2}$$



$$E^{\perp}_{net} = \int dE \cos\theta$$

$$\cos\theta = \frac{z}{\sqrt{x^2+z^2}}$$

$\underline{E}$ field

4/ $$\therefore E^{\perp}_{net} = \frac{\lambda}{2\pi\varepsilon_0}\int_{-\infty}^{0} \frac{z\,dx}{x^2+z^2}$$

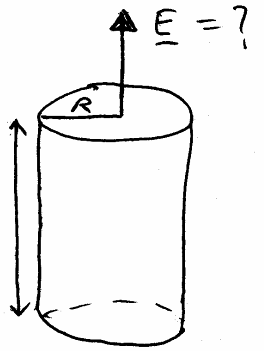
$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\tan^{-1}\left(\frac{x}{z}\right)\right]_{-\infty}^{0}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\cdot\frac{\pi}{2}$$

$$= \frac{\lambda}{4\varepsilon_0}$$

$\underset{\sim}{E}$ field

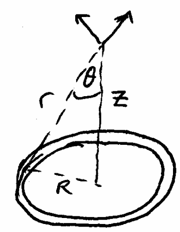
$\underline{S}$



hollow cylinder
surface charge
density $= \sigma$

divide into rings

ring of charge



$$E = \int dE \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2} \cdot \frac{z}{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda\, dl\, z}{r^3}$$

$$= \frac{\lambda z}{4\pi\varepsilon_0} \int \frac{dl}{(z^2+R^2)^{3/2}}$$

$$= \frac{\lambda z \cdot 2\pi R}{4\pi\varepsilon_0 \left(z^2+R^2\right)^{3/2}}$$

$\underline{E}$ field

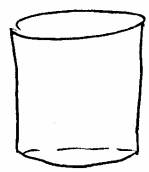
5/

$$E_{ring} = \frac{\lambda z R}{2\varepsilon_0 (z^2+R^2)^{3/2}}$$

$$q_{ring} = \lambda \cdot 2\pi R$$

$$\therefore\ E_{ring} = \frac{1}{4\pi\varepsilon_0} \frac{q_{ring}\ z}{(z^2+R^2)^{3/2}}$$

Now, cylinder



$\downarrow z$ (0 to L)

$$q_{ring} = \sigma\, dz \cdot 2\pi R$$

$$E_{total} = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma \cdot 2\pi R \cdot z\, dz}{(z^2+R^2)^{3/2}}$$

$$= \frac{\sigma R}{2\varepsilon_0} \int \frac{z\, dz}{(z^2+R^2)^{3/2}}$$

$$= \frac{\sigma R}{2\varepsilon_0} \left| \left[\frac{1}{\sqrt{z^2+R^2}} \right]_0^L \right|$$

$$= \frac{\sigma R}{2\varepsilon_0} \left[\frac{1}{R} - \frac{1}{\sqrt{R^2+L^2}} \right]$$

$$= \frac{\sigma}{2\varepsilon_0} \left[1 - \frac{R}{\sqrt{R^2+L^2}} \right]$$

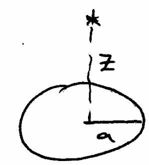
$\underline{E}$ field

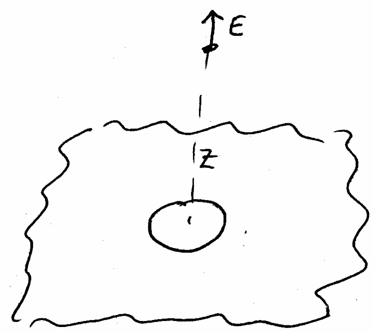
6/ E (∞ plane sheet)

$$= \frac{\sigma}{2\varepsilon_0}$$

$$E \text{ (disc)} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{a^2+z^2}}\right]$$

<u>lectures</u>





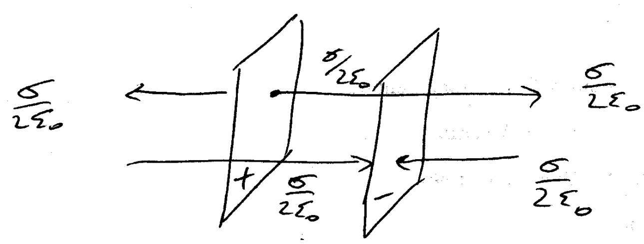
hole in sheet

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{a^2+z^2}}\right]$$

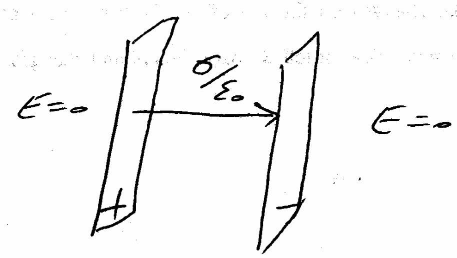
$$= \frac{\sigma z}{2\varepsilon_0\sqrt{a^2+z^2}}$$

$\underline{E}$ field

7 lectures



∴ fields are



$\underline{E}$ field

8/ non-conducting sphere - radius R

$$\rho = \rho_o \left[1 - \left(\frac{r}{R}\right)^2\right]$$

a/. $Q_{total} = ?$

$$\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{R} \rho \, d\tau$$

$$= \iiint \rho_o \left[1 - \left(\frac{r}{R}\right)^2\right] \cdot r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= 2\pi\rho_o \int_\theta \int_r \left\{ r^2 \sin\theta - \frac{r^4 \sin\theta}{R^2} \right\} dr \, d\theta$$

$$\int_{\theta=0}^{\pi} \sin\theta \, d\theta = -\cos\theta \Big|_0^\pi = 2$$

so $Q_{total} = 4\pi\rho_o \int_r \left\{ r^2 - \frac{r^4}{R^2} \right\} dr$

$$= 4\pi\rho_o \left[\frac{1}{3} R^3 - \frac{1}{5} \frac{R^5}{R^2} \right]$$

$$= 4\pi\rho_o \cdot \frac{2R^3}{15} = \frac{8\pi\rho_o R^3}{15}$$

$\underline{E}$ field

8/ (b) $E(r)$ outside sphere

$$= \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{8\pi\rho_0 R^3}{15 r^2}$$

$$= \frac{2\rho_0 R^3}{15\varepsilon_0 r^2} \hat{\underline{r}}$$

(c) $E(r)$ inside sphere

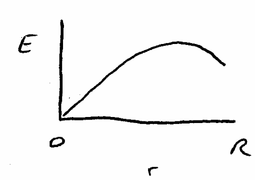
$$\oint \underline{E} \cdot d\underline{a} = \frac{Q_{encl}}{\varepsilon_0} \qquad \text{Gauss}$$

$$4\pi r^2 E = \frac{Q_{encl}}{\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}\left[\frac{1}{3}r^3 - \frac{1}{5}\frac{r^5}{R^2}\right] \cdot 4\pi\rho_0 \quad \text{(from part (a)}$$

$$\therefore E = \frac{4\pi\rho_0\left[\frac{1}{3}r^3 - \frac{1}{5}\frac{r^5}{R^2}\right]}{4\pi\varepsilon_0 r^2}$$

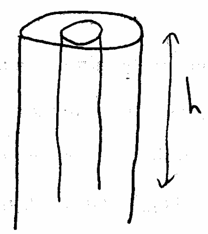
$$= \frac{\rho_0}{\varepsilon_0}\left[\frac{r}{3} - \frac{r^3}{5R^2}\right]$$



$\underline{E}$ field

$\underline{9}$ Long cylinder

$\rho_v = Ar$ $\quad$ A = constant
r = distance from axis



$E(r)$ inside cylinder $= ?$

Gaussian surface

$$\int \underline{E} \cdot d\underline{a} = \frac{Q_{encl}}{\varepsilon_0}$$

$$E \cdot 2\pi r h = \frac{Q_{encl}}{\varepsilon_0}$$ central part.

$$\left[\text{end-caps} = 0 \qquad \underline{E} \perp \underline{A} \right]$$

$$Q_{encl} = \int_{z=0}^{h} \int_{\phi=0}^{2\pi} \int_{r} \rho \, d\tau$$

$$= \iiint Ar \cdot r \, dr \, d\phi \, dz$$

$$= A \cdot h \cdot 2\pi \int r^2 dr$$

$$= \frac{2\pi A h r^3}{3}$$

$\underline{E}$ field

$\underline{9}$

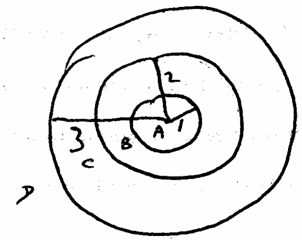
$$\therefore \; 2\pi r h \, E = \frac{2\pi A h r^3}{3\varepsilon_0}$$

$$E(r) = \frac{2\pi A h r^3}{2\pi r h \cdot 3\varepsilon_0}$$

$$= \frac{A r^2}{3\varepsilon_0} \; \underline{\hat{r}}$$

$\underline{E}$

$\underline{10}$ 3 concentric spherical shells



radii 1, 2, 3 m
charges 3, −1, −2 nC

$$E_A = 0 \quad \because \quad Q_{encl} = 0$$

$$E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{3nC}{r^2} = \frac{27}{r^2} \quad Vm^{-1}$$

$$E_C = \frac{1}{4\pi\varepsilon_0} \frac{2nC}{r^2} = \frac{18}{r^2} \quad Vm^{-1}$$

$$E_D = 0 \quad \because \quad Q_{encl} = 0$$