Magnetic Materials

1. Solenoid 500 turns/metre

wound on Fe. $\mu_r = 800$

$i = 0.5\,A$

$H = ni = 500 \times 0.5 = 250\ A/m$

$B = \mu_r \mu_0 n i = 800 \times 4\pi \times 10^{-7} \times 500 \times 0.5$

$= 0.25\ T$

$M = ?$ $\quad B = \mu_0 (H + M)$

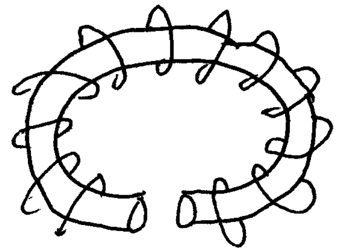
$\therefore M = \dfrac{B}{\mu_0} - H$

$= (\mu_r - 1) . ni$

$= 799 \times H$

$= 799 \times 250 = 199750\ A m^{-1}$

2/



toroid

$$l_{Fe\ path} = 50\ \text{cm} \quad : \quad l_{gap} = 1\ \text{mm}$$

$$B \text{ in } Fe = 1.5\ T$$

$$\mu_r = 5000$$

a) $B_{gap} = ?$

$$\phi = B_{gap} A_{gap} = B_{Fe} A_{Fe}$$

$A_{gap} = A_{Fe}$ (cross-sections)

$$\therefore B_{gap} = B_{Fe} = 1.5\ T$$

b) $H_{Fe} = ?$

$$B_{Fe} = \mu_r \mu_0 H_{Fe}$$

$$\therefore H_{Fe} = \frac{1.5\ T}{5000\ \mu_0}$$

$$= 239\ \text{Am}^{-1}$$

2 c

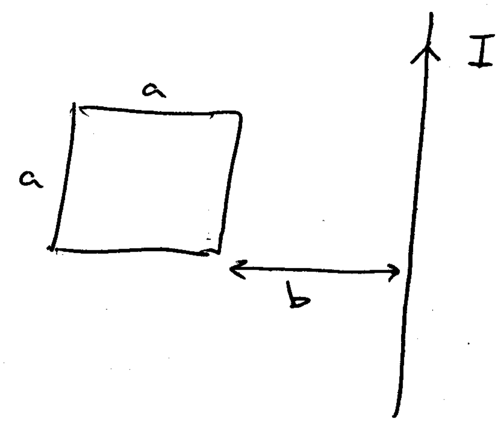
$$H_{gap} = ?$$

$$B_{gap} = \mu_0 H_{gap} = B_{Fe} = 1.5\,T$$

$$\therefore\ H_{gap} = \frac{1.5}{\mu_0} = 1.2 \times 10^6\ Am^{-1}$$

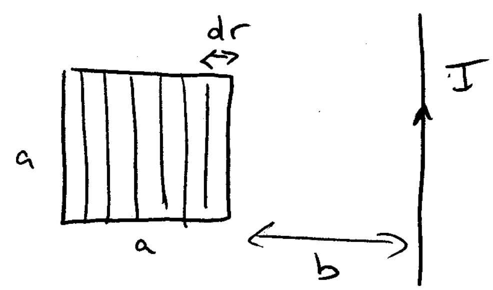
Induction

3. Square loop near a current-carrying wire



ϕ through loop = ?

divide loop



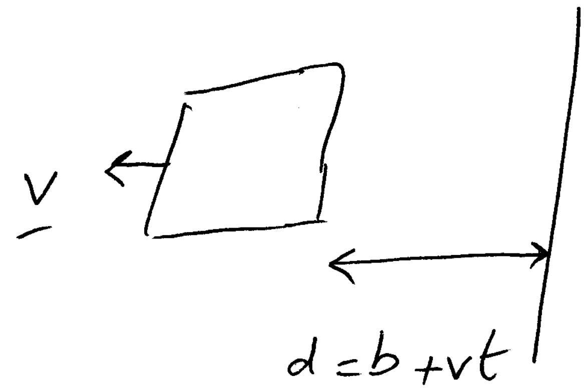
$$B = \frac{\mu_0 I}{2\pi r}$$

$d\phi$ through a slice $= B\,dA$

$$= \frac{\mu_0 I}{2\pi r} \cdot a\,dr$$

$$\therefore \phi = \int d\phi = \frac{\mu_0 I a}{2\pi} \int_b^{b+a} \frac{dr}{r} = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{b+a}{b}\right)$$

Pull loop away from wire



$\mathcal{E} = ?$

$$\phi = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{b+vt+a}{b+vt}\right)$$

$$\mathcal{E} = -\dot{\phi} = -\frac{\mu_0 I a}{2\pi} \frac{d}{dt}\left[\ln\left(\frac{b+vt+a}{b+vt}\right)\right]$$

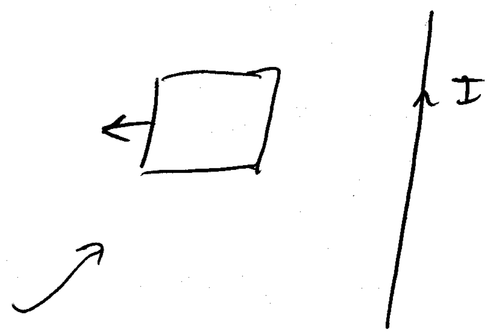
$$= -\frac{\mu_0 I a}{2\pi}\left(\frac{b+vt}{b+vt+a}\right)\frac{d}{dt}\left[\frac{b+vt+a}{b+vt}\right]$$

$$= -\frac{\mu_0 I a}{2\pi}\left(\frac{b+vt}{b+vt+a}\right)\left[\frac{(b+vt)v - v(b+vt+a)}{(b+vt)^2}\right]$$

$$= -\frac{\mu_0 I a}{2\pi}\left[\frac{-av}{(b+vt+a)(b+vt)}\right]$$

$$= \frac{\mu_0 I a^2 v}{2\pi (b+vt+a)(b+vt)}$$

3. Dir^n of induced current



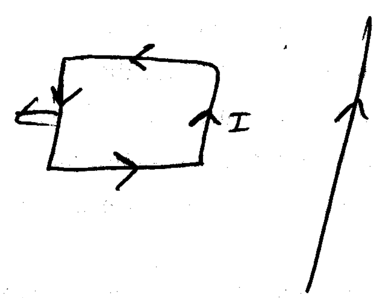
getting further from wire

$\therefore$ B $\downarrow$ $\therefore$ ϕ through loop $\downarrow$

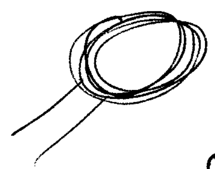
B due to $\uparrow I$ is $\odot$

$\therefore$ want $I_{induced}$ s.t. B_{loop} is $\odot$

$\therefore$



B4 Search coil



diameter = 2 cm

if stray field (to be detected) has an amplitude of 1 mT at 50 Hz, how many turns in the search coil to give 1 mV RMS

$$B = 1\,\text{mT} \sin \omega t$$

$$= 10^{-3} \sin 100\pi t$$

$$\phi = BA = 10^{-3} \sin(100\pi t) \cdot \pi r^2$$

$$-\dot{\phi} = -\pi r^2 \times 10^{-3} \times 100\pi \cos(100\pi t) \times n$$

$$= \mathcal{E}$$

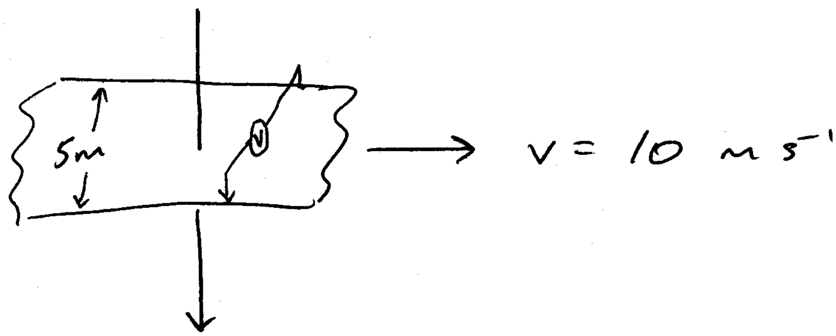
$$\mathcal{E}_{RMS} = \frac{\pi r^2 \times 10^{-3} \times 100\pi \times n}{\sqrt{2}} = 1 \times 10^{-3}\ \text{V}$$

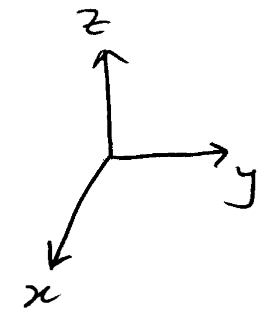
$$\therefore \quad n = \frac{\sqrt{2}}{\pi^2 \times r^2 \times 100} \qquad r = 0.01\,\text{m}$$

$$= 14.3$$

i.e. 14 turns

#5 Al sheet





$\underline{B} = 2.5 \times 10^{-5}$ T

voltage induced between the edges of the sheet = ?

F on a charge q in the sheet

$= q\underline{v} \times \underline{B}$ $\qquad \underline{v} \perp \underline{B}$

$\therefore F = qvB$ in x dir^{n}

$E = F/q = vB$

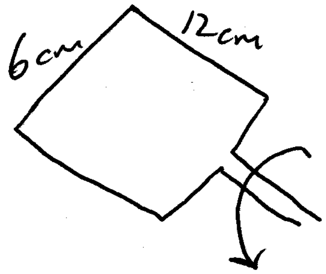
$\mathcal{E} = \int \underline{E} \cdot d\underline{l} = \int_0^5 vB\,dx$

$= vB\,x\Big]_0^5$

$= 10 \times 2.5 \times 10^{-5} \times 5$

$= 1.25$ mV

5.

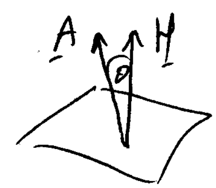


H normal to loop axis = $40\ A\,m^{-1}$

$\mathcal{E}_{RMS}$ induced in loop = ?

$$\phi = \underline{B} \cdot \underline{A} = \mu_0 \underline{H} \cdot \underline{A}$$

$$= \mu_0 \times 40 \times 0.06 \times 0.12 \times \cos\theta$$



$$\theta = \omega t = 2\pi\nu t$$

$$-\dot{\phi} = \mathcal{E} = -\mu_0 \times 40 \times 0.06 \times 0.12 \times 2\pi\nu \sin(2\pi\nu t)$$

$$\therefore |\mathcal{E}_{RMS}| = \frac{\mu_0 \times 40 \times 0.06 \times 0.12 \times 2\pi\nu}{\sqrt{2}}$$

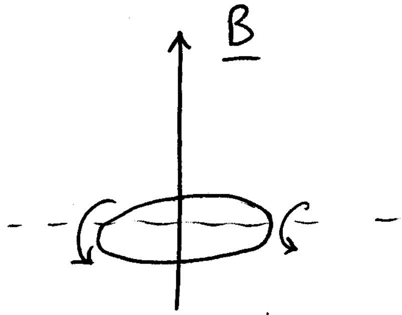
$$\nu = \frac{8000}{60}$$

$$\text{So } \mathcal{E}_{RMS} = \frac{\mu_0 \times 40 \times 0.06 \times 0.12 \times 2\pi \times 8000}{60\sqrt{2}}$$

$$= 2.14 \times 10^{-4}\ V$$

7a

6.7



10 cm diameter, single-turn coil. Spins at 10000 rpm

$I_{induced} = 2A$ rms

$R_{Loop} = 0.1\,\Omega$

a) average Power to spin the loop

$= I_{RMS}^2 R = 2^2 \times 0.1 = 0.4\,W$

b) average torque on Loop

$\underline{\tau} = \underset{\sim}{\mu} \times \underline{B}$

to get B we need $\mathcal{E}$

$\mathcal{E} = IR = 0.2\,V$ rms

$\therefore \quad \mathcal{E} = 0.2\sqrt{2}\,\sin(2\pi\nu t)$

$\nu = \frac{10000}{60}$

Maxwell's Eq^{n}

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$$\underset{\sim}{E} = 10 \cos\left[\omega\left(t - \frac{z}{c}\right)\right] \hat{\underset{\sim}{i}} \quad V m^{-1}$$

$$\underset{\sim}{H} = \; ?$$

$$\underline{\nabla} \times \underline{E} = -\frac{\partial \underset{\sim}{B}}{\partial t}$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E & 0 & 0 \end{vmatrix}$$

$$= -\hat{\underset{\sim}{j}}\left(-\frac{\partial \underset{\sim}{E}}{\partial z}\right) = +\hat{\underline{j}} \cdot 10 \frac{\omega}{c} \sin\left[\omega\left(t - \frac{z}{c}\right)\right]$$

$$\therefore \frac{\partial \underset{\sim}{B}}{\partial t} = -\frac{10\omega}{c} \sin\left[\omega\left(t - \frac{z}{c}\right)\right] \hat{\underline{j}}$$

$$\therefore \underset{\sim}{B} = \frac{10\omega}{c\omega} \cos\left[\omega\left(t - \frac{z}{c}\right)\right] \hat{\underline{j}}$$

$$= \frac{10}{c} \cos\left[\omega\left(t - \frac{z}{c}\right)\right] \hat{\underline{j}}$$

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$$\underset{\sim}{H} = \frac{\underset{\sim}{B}}{\mu_0}$$

$$= \frac{10}{\mu_0 c} \cos\left[\omega\left(t - \frac{z}{c}\right)\right] \hat{\underset{\sim}{j}} \quad Am^{-1}$$

$$= 0{\cdot}0265 \cos\left[\omega\left(t - \frac{z}{c}\right)\right] \hat{\underline{j}} \quad Am^{-1}$$

Poynting Vector

$$\underset{\sim}{S} = \frac{1}{\mu_0} \underset{\sim}{E} \times \underline{B}$$

$$= \frac{1}{\mu_0} \begin{vmatrix} \hat{\underline{i}} & \hat{\underline{j}} & \hat{\underline{k}} \\ E & 0 & 0 \\ 0 & B & 0 \end{vmatrix}$$

$$= \frac{1}{\mu_0} EB \hat{\underline{k}} = \frac{1}{\mu_0} 10 \cos\left[\omega\left(t - \frac{z}{c}\right)\right] \cdot \frac{10}{c} \cos\left[\omega\left(t - \frac{z}{c}\right)\right] \hat{\underline{k}}$$

$$= \frac{100}{\mu_0 c} \cos^2\left[\omega\left(t - \frac{z}{c}\right)\right] \hat{\underline{k}}$$

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$$\underline{E} = 250 \cos\left[10^9 t - 5x\right] \hat{\underline{k}} \ \ V m^{-1}$$

$\mu = \mu_0$

a) dir^n of travel = $+x$

Look at max in curve

i.e. when

$$10^9 t - 5x = 0$$

at $t = 0$, $x = 0$
$t > 0$, $x > 0$ $\Big\}$ $\therefore$ $+x$ motion

b) $v = \omega / k = 10^9 / 5$

$$= 2 \times 10^8 \ m s^{-1}$$

c) $\underline{H} = ?$

9. $$\underline{\nabla} \times \underset{\sim}{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$= \begin{vmatrix} \hat{\underline{i}} & \hat{\underline{j}} & \hat{\underline{k}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & E \end{vmatrix}$$

$$= -\hat{\underline{j}} \frac{\partial E}{\partial x} = -\hat{\underline{j}} \times 250 \times 5 \sin[10^9 t - 5x]$$

$$\therefore \frac{\partial \underline{B}}{\partial t} = 1250 \sin[10^9 t - 5x]\, \hat{\underline{j}}$$

$$\therefore \underline{B} = -\frac{1250}{10^9} \cos[10^9 t - 5x]\, \hat{\underline{j}}$$

$$= \mu_0 \underline{H}$$

$$\therefore \underline{H} = \frac{-1250}{10^9 \mu_0} \cos[10^9 t - 5x]\, \hat{\underline{j}}$$

$$= -0.995 \cos[10^9 t - 5x]\, \hat{\underline{j}} \quad A\,m^{-1}$$

10

4/ $\underset{\sim}{E} = 100 \cos(2\pi \times 10^8 t - 3y)\,\hat{\underline{i}}$ Vm^{-1}

a). relative permittivity of medium = ?

$[\mu = \mu_0]$

$$\therefore \frac{c}{v} = \sqrt{\frac{\varepsilon}{\varepsilon_0}} \qquad v = \frac{1}{\sqrt{\varepsilon\mu}}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$\therefore v = \omega/k = \frac{2\pi \times 10^8}{3}$$

$$\therefore \frac{c}{v} = \frac{3 \times 10^8}{2\pi \times 10^8/3} = \frac{9}{2\pi} = 1.4324$$

$$\therefore \frac{\varepsilon}{\varepsilon_0} = 1.4324^2 = 2.05$$

b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{3} = 2.09\ m$

c) average power density in the wavefront

$$\underline{S} = \frac{1}{\mu_0} \underline{E} \times \underline{B}$$

4. Get $\underline{B}$

$$\underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$= \begin{vmatrix} \hat{\underline{i}} & \hat{\underline{j}} & \hat{\underline{k}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E & 0 & 0 \end{vmatrix}$$

$$= -\hat{\underline{k}} \frac{\partial E}{\partial y}$$

$$= -300 \sin(2\pi \times 10^8 t - 3y) \hat{\underline{k}}$$

$$= -\frac{\partial \underline{B}}{\partial t}$$

$$\therefore \underline{B} = \frac{-300}{2\pi \times 10^8} \cos(2\pi \times 10^8 t - 3y) \hat{\underline{k}}$$

$$|\underline{S}|_{average} = \frac{1}{\mu_0} \langle E B \rangle$$

$$= \frac{300 \times 100}{2\pi \times 10^8 \mu_0} \left\langle \cos^2 [2\pi \times 10^8 t - 3y] \right\rangle$$

$$= \frac{300 \times 100}{2\pi \times 10^8 \mu_0} \times \frac{1}{2}$$

$$= 19 \ W m^{-2}$$