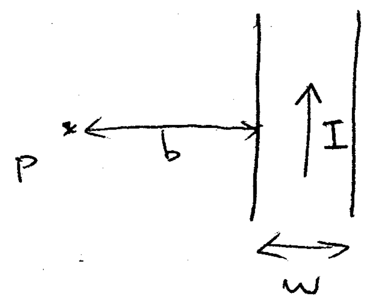
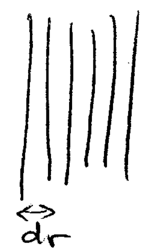
Problem Sheet 5 — Magnetic Fields

1/ Long thin flat strip



B at P = ?

divide strip into infinitesimals



$$B = \int \frac{\mu_0 I_{net}}{2\pi r}$$

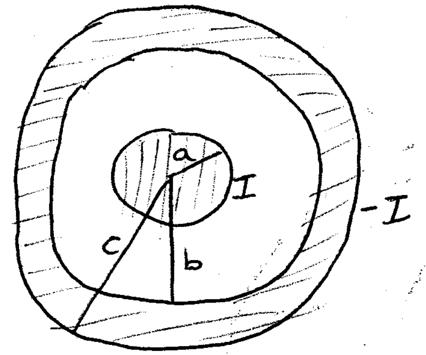
$$I_{net} = I_{strip} = \frac{I\,dr}{W}$$

$$\therefore B = \frac{\mu_0 I}{2\pi W} \int_b^{b+W} \frac{dr}{r}$$

$$= \frac{\mu_0 I}{2\pi W} \ln\left(\frac{b+W}{b}\right)$$

$$= \frac{\mu_0 I}{2\pi W} \ln\left(1 + \frac{W}{b}\right)$$

2/



co-axial

$$\oint \underline{B} \cdot d\underline{s} = \mu_0 I_{encl}$$

$r < a$

$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0 I}{2\pi r}\left(\frac{\pi r^2}{\pi a^2}\right)$$

$$= \frac{\mu_0 I r}{2\pi a^2}$$

$a < r < b$ $\therefore$ just inner

$$B = \frac{\mu_0 I}{2\pi r}$$

$b < r < c$

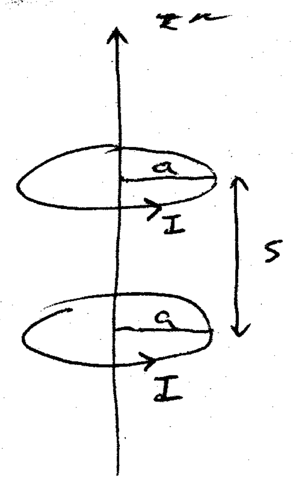
$$I_{encl} = I - I\,\frac{\pi(r^2-b^2)}{\pi(c^2-b^2)}$$

$$= I\left(\frac{c^2-r^2}{c^2-b^2}\right)$$

$$\therefore B = \frac{\mu_0 I}{2\pi r}\left(\frac{c^2-r^2}{c^2-b^2}\right)$$

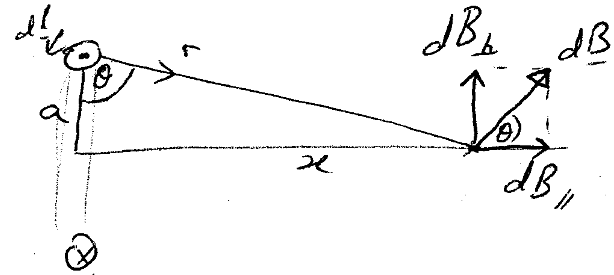
$r > c$ $\quad I_{net} = 0 \quad \therefore \quad B = 0$

3/



B due to a single loop

single loop



only $dB_{\parallel}$ contributes

$$dB = \frac{\mu_0 i}{4\pi} \frac{d\underline{l} \times \hat{\underline{r}}}{r^2}$$

$$= \frac{\mu_0 i}{4\pi r^2} dl \sin 90^\circ = \frac{\mu_0 i\, dl}{4\pi r^2}$$

$$dB_{\parallel} = dB \cos\theta = \frac{\mu_0 i \cos\theta\, dl}{4\pi r^2}$$

$$r = \sqrt{a^2+x^2} \qquad \cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+x^2}}$$

$$= dB_{\parallel} = \frac{\mu_0 i a}{4\pi (a^2+x^2)^{3/2}} dl$$

$$B = \int dB_{\parallel} = \frac{\mu_0 i a}{4\pi (a^2+x^2)^{3/2}} \int dl$$

$$= \frac{\mu_0 i a}{4\pi (a^2+x^2)^{3/2}} \cdot 2\pi a$$

3

So
$$B = \frac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}}$$

we want
$$\frac{d^2B}{dx^2} = 0$$

$$\frac{dB}{dx} = \frac{\mu_0 I a^2}{2} \frac{d}{dx}\left[a^2+x^2\right]^{-3/2}$$

$$= \frac{\mu_0 I a^2}{2} \cdot -\frac{3}{2} \cdot 2x \cdot (a^2+x^2)^{-5/2}$$

$$= \frac{-3\mu_0 I a^2}{2} \cdot \frac{x}{(a^2+x^2)^{5/2}}$$

$$\frac{d^2B}{dx^2} = \frac{-3\mu_0 I a^2}{2}\left\{\frac{(a^2+x^2)^{5/2} - x \cdot \frac{5}{2} \cdot 2x(a^2+x^2)^{3/2}}{(a^2+x^2)^5}\right\}$$

$= 0$

$$\therefore (a^2+x^2)^{5/2} = 5x^2(a^2+x^2)^{3/2}$$

$$(a^2+x^2) = 5x^2$$

$$x = \frac{a}{2}$$

3/

B for 2 loops is

$$\frac{\mu_0 I a^2}{(a^2+x^2)^{3/2}} \quad \text{with} \quad x = a/2$$

$$= \frac{\mu_0 I a^2}{\left(a^2 + \frac{a^2}{4}\right)^{3/2}}$$

$$= \frac{8 \mu_0 I}{a \cdot 5^{3/2}}$$

cancel B_{earth} ($= 50 \mu T$). Each coil has 10 turns and radius $= 0.5$ m.

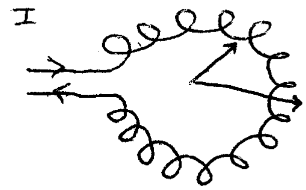
$I = ?$

$$\frac{8 \times 4\pi \times 10^{-7} \times 10 \times I}{0.5 \times 5^{3/2}} = 50 \times 10^{-6}$$

$\Rightarrow I = 2.78$ A

4

torus



500 turns

$r_{inner} = 10\,cm$

$r_{outer} = 12\,cm$

$I = 5\,mA$

$$B = \frac{\mu_0 N I}{2\pi r} \qquad r = 11\,cm$$

$$= \frac{4\pi \times 10^{-7} \times 500 \times 5 \times 10^{-3}}{2\pi \times 0.11}$$

$$= 4.54 \times 10^{-6}\,T$$

5/

Solenoid $L = 10\text{ cm}$

$r = 1\text{cm}$

$N = 10000$ turns

I to cancel $B_{earth} = ?$

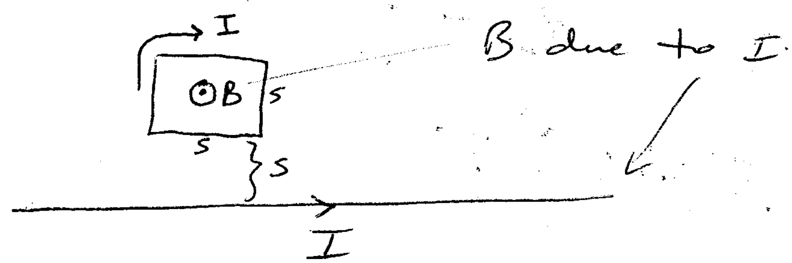
$$B = \mu_0 n I$$

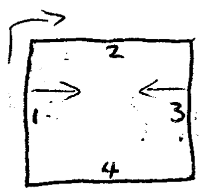
$$= 4\pi\times10^{-7} \times \frac{10000}{0.10} \times I = 50\times10^{-6}$$

$$\Rightarrow I = 3.98\times10^{-4}\,A$$

Magnetic forces

6





F_1 and F_3 cancel (symmetry).

$$B \text{ on arm } \#2 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I}{2\pi \cdot 2s}$$

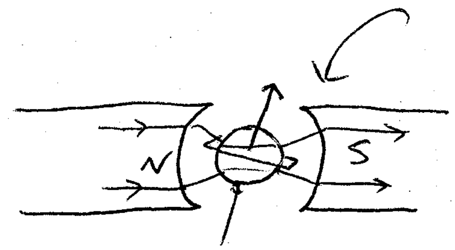
$$B \text{ on arm } \#4 = \frac{\mu_0 I}{2\pi s}$$

$$F_2 = I L B_2 = I \cdot s \cdot \frac{\mu_0 I}{4\pi s} = \frac{\mu_0 I^2}{4\pi} \quad \boxed{\downarrow}$$

$$F_4 = I L B_4 = \frac{\mu_0 I^2}{2\pi} \quad \boxed{\uparrow}$$

$$\therefore F_{net} = F_4 - F_2 = \frac{\mu_0 I^2}{4\pi} \quad \boxed{\uparrow}$$

#7



torque $= 2\times10^{-6}$ Nm / degree

coil has 50 turns, $0.016\,m \times 0.025\,m$

$\perp$ to page

I to deflect needle by 45°

$$\underline{m} = I_{total}\,\underline{A}$$

$$= N I \underset{\sim}{A} \qquad \underset{\sim}{m} \perp \underline{B}$$

$$\therefore \underset{\sim}{\tau} = \underset{\sim}{m} \times \underset{\sim}{B} = N I \underset{\sim}{A} \times \underset{\sim}{B}$$

$$\tau = N I A B$$

$$I = \frac{\tau}{NAB} = \frac{2\times10^{-6}\times 45}{50\times 0.016\times0.025\times0.8}$$

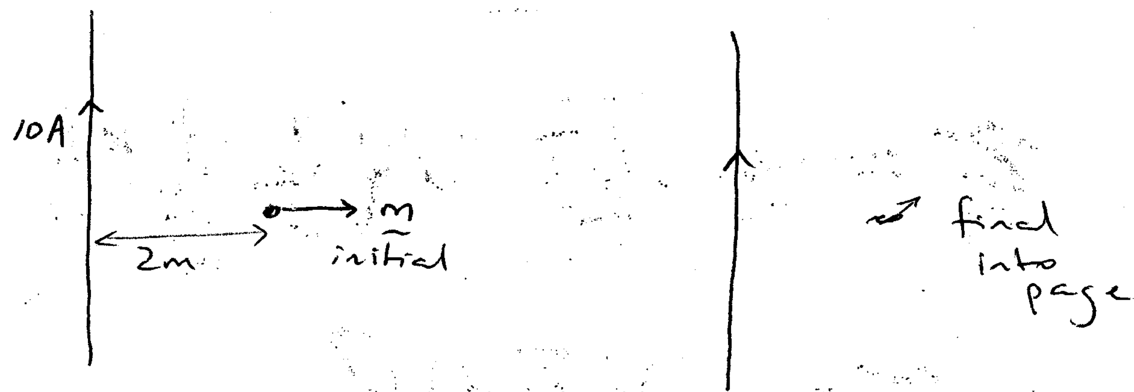
$$= 5.62\times10^{-3}\ A$$

8/

mag dipole - initially

$$\underset{\sim}{m} = 5\,\hat{\underline{r}} \quad \mu A \cdot m^2$$

2 m from an ∞ conductor along z



$$\underset{\sim}{\tau}_i = \underset{\sim}{m}_i \times \underset{\sim}{B}$$

$$\tau = mB \sin 90$$

$$= mB$$

$$= 5\times10^{-6} \times \frac{\mu_0 I}{2\pi r}$$

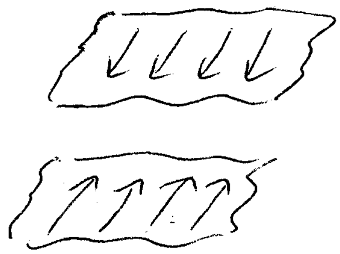
$$= \frac{5\times10^{-6} \times 4\pi\times10^{-7}\times10}{2\pi\times2}$$

$$= 5\times10^{-12}\,\mathrm{Nm}\ \hat{\underset{\sim}{z}}$$

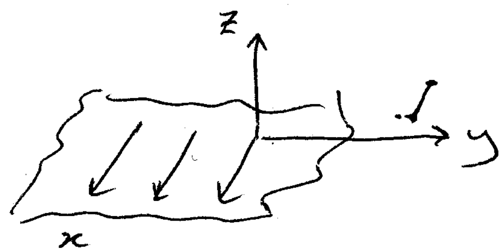
$$E = -\cancel{\mu}\ \underline{m}\cdot\underline{B}$$

$$\tau_{final} = 0 \quad \because \quad \underset{\sim}{m} \parallel \underline{B}$$

9 2 ∞ parallel conducting sheets 1mm apart carry surface currents $K = 5\ \mathrm{A\,m^{-1}}$ in opposite dir^{-s}. Force between sheets $= ?$

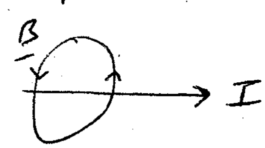


Calculate B due to one sheet



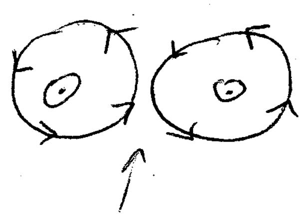
$\underline{B}$ can't have an x component

$\therefore \underline{B}$ is in plane $\perp$ I



$\underline{B}$ can't have a z component $\because \infty$ sheets

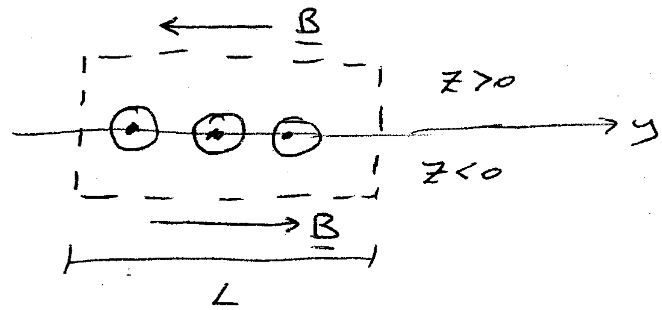
imagine current strips

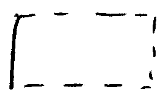
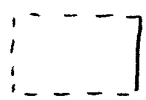


Cancellation

$\therefore$ $\underline{B}$ is in $\pm y$ dir^{n}

rectangular Ampèrian loop



 and  give zero contrib^{n} to $\oint \underline{B} \cdot d\underline{l}$ $\because$ $\underline{B} \perp d\underline{l}$

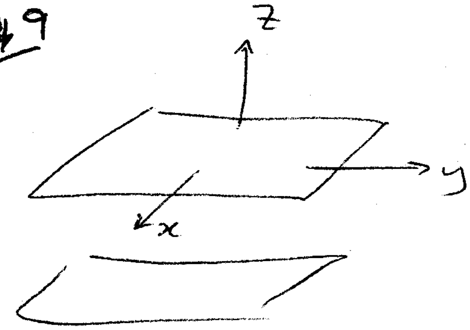
so $\oint \underline{B} \cdot d\underline{l}$

$$= 2BL = \mu_0 I_{encl}$$

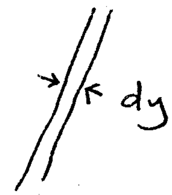
$$= \mu_0 K L$$

$$\therefore \; B = \frac{\mu_0 K}{2}$$

$$\underline{B} = \begin{cases} +\dfrac{\mu_0 K}{2}\,\hat{\underset{\sim}{j}} & \text{for } z<0 \\[2ex] -\dfrac{\mu_0 K}{2}\,\hat{\underset{\sim}{j}} & \text{for } z>0 \end{cases}$$



$$\underline{F} = \int I \, d\underline{l} \times \underline{B}$$



$$I = K \, dy$$

$$dl = dx$$

$$\therefore \underset{\sim}{F} = \int_0^1 \int_0^1 K \cdot \frac{\mu_0 K}{2} \, dx \, dy$$

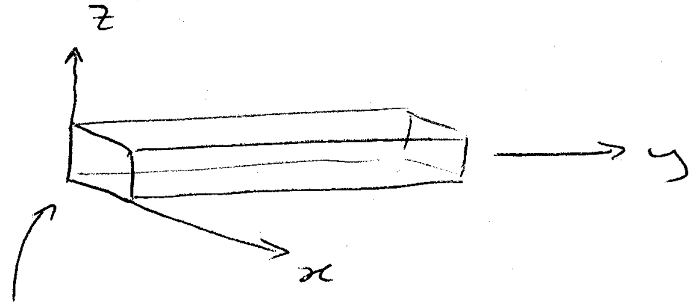
$$= \frac{\mu_0 K^2}{2}$$

$\int_0^1$ $\because$ we want force per unit area

$$= \frac{4\pi \times 10^{-7} \times 5^2}{2}$$

$$= 1.57 \times 10^{-5} \ \mathrm{N \, m^{-2}}$$

10 ~~a~~



$0 \le x \le 0.1\ \text{m}$

$0 \le z \le 0.2\ \text{m}$

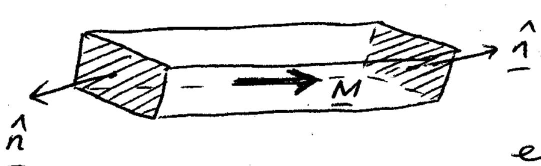
$$\underset{\sim}{M} = 3x\,\hat{\underset{\sim}{j}}\ \ \text{A m}^{-1}$$

$$\underset{\sim}{J}_b = \underline{\nabla} \times \underset{\sim}{M}$$

$$= \begin{vmatrix} \hat{\underline{i}} & \hat{\underline{j}} & \hat{\underline{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \partial/\partial z \\ 0 & 3x & 0 \end{vmatrix}$$

$$= 3\,\hat{\underline{k}}\ \ \text{A m}^{-2}$$

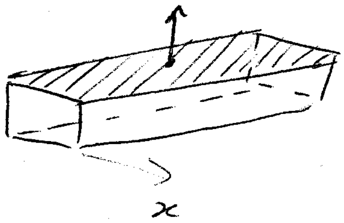
$$\underset{\sim}{K}_b = \underset{\sim}{M} \times \hat{\underline{n}}$$



end-bits $\rightarrow 0$

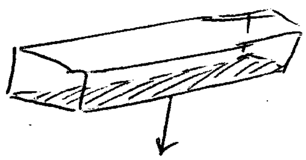
$$\underset{\sim}{M} \times \hat{\underset{\sim}{n}} = 0$$

10



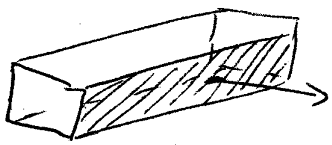
$$\hat{n} = \hat{k}$$

$$K_b = 3x\hat{j} \times \hat{k} = 3x\hat{i}$$



$$\hat{n} = -\hat{k}$$

$$\Rightarrow K_b = -3x\hat{i}$$

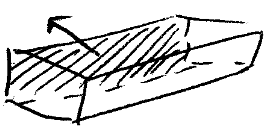


$$\hat{n} = \hat{i}$$

$$K_b = 3x\hat{j} \times \hat{i} = -3x\hat{k}$$

$$x = 0.1$$

$$\therefore K_b = -0.3\,\hat{k}\ \mathrm{Am}^{-1}$$



$$\hat{n} = -\hat{i}$$

$$K_b = 3x\hat{j} \times (-\hat{i})$$

$$= 0 \quad \because x = 0$$