**THE UNIVERSITY OF NEW SOUTH WALES**
**SCHOOL OF PHYSICS**

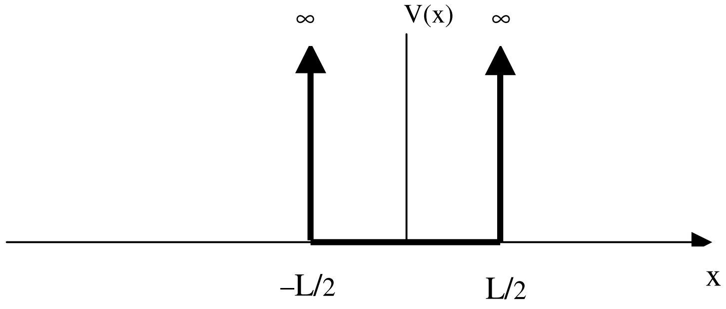
# PHYS2040 QUANTUM PHYSICS
# TUTORIAL PROBLEMS

These problems are intended to illustrate and reinforce the course material. It is important to work through these problems, or attempt to. Some will be done in class as examples. The problems marked (*) are more difficult and are included as a challenge for the better students.

1. In an X-ray tube electrons of energy 10 keV are incident on a metal target. Calculate the shortest wavelength of X-rays produced. [1.24 x $10^{-10}$ m]
2. Solar radiation falls on the earth's surface with an average energy flux of 1400 J $m^{-2}$ $s^{-1}$. Assuming an average wavelength of 550 nm, how many photons is this ? [3.9 x $10^{21}$ photons $m^{-2}$ $s^{-1}$]
3. White light (wavelength range 400-700 nm) falls on a potassium surface whose work function is 2.2 eV. What is the speed of the most energetic photoelectrons ? [5.65 x $10^5$ m $s^{-1}$]
4. In a photoelectric experiment using a sodium cathode, the stopping potential is measured to be 1.85 V for $\lambda$=300 nm and 0.82 V for $\lambda$ =400 nm. Determine the work function of sodium and also a value for Planck's constant from these data. [2.27 eV, 6.59 x $10^{-34}$ Js]
5. A photon of frequency f collides with a stationary electron and is scattered through an angle $\theta$. The electron recoils with speed v at an angle $\phi$. Draw a sketch of the scattering process and write down equations expressing the conservation of energy and momentum.
6. * Show that it is impossible for a free electron to absorb a photon. How is it then possible for a photon to eject an electron from an atom ?
7. Calculate the average de Broglie wavelength of a hydrogen molecule at room temperature. [1.04 Å].
8. A beam of 120 eV electrons is incident on a nickel crystal with lattice spacing of 2.15 Å. Calculate the de Broglie wavelength of the electrons ? Calculate the angle between the incident beam and the 2nd order diffracted beam. [1.12 Å , 117.2°].
9. Show that for non-relativistic electrons the group velocity for de Broglie waves is equal to the particle velocity.
10. Use the Heisenberg Uncertainty Principle to estimate the minimum kinetic energy that an electron must have to be confined within a nucleus of radius $10^{-14}$ m [~ 9.4 MeV].
11. A particle is confined to the region $0<x<1$ on the x-axis and has a probability density $P(x) = 1-\cos(2\pi x)$. Calculate the probability of finding the particle in the regions (i) $0<x<0.01$, (ii) $0.5 < x<0.51$ [6.58 x $10^{-6}$, 0.02]

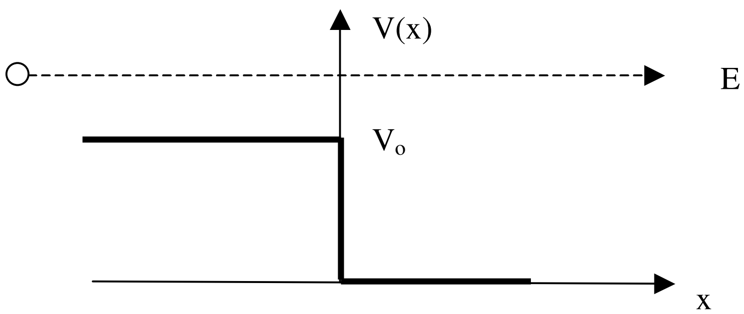
12. * Consider a wavefunction of the form $\Psi(x,t)=\sum_n a_n\,\phi_n(x)e^{-iE_nt/\hbar}$ where $\{E_n\}$ are the energies of stationary states and $\{\phi_n\}$ the corresponding state functions. Show that $\Psi(x,t)$ satisfies the Schrödinger equation.

13. Solve the time-independent Schrödinger equation for the infinite square well in the alternative form shown and show that the same result is obtained for the allowed energies as for the usual form of V(x)




14. * An electron is located in a rectangular finite-potential well of width 4 Å and depth 5 eV. Determine the number and energies (in eV) of the bound states. NB: this is a 'transcendental' problem so you cannot solve it analytically. You will have to 'solve' your equations using either graphical or computer-numerical methods. [2, 1.11 eV and 3.98 eV]

15. A particle is incident from the left onto a potential step of the form shown. Write down the form of the wavefunction in the two regions and the connecting relations at the boundary.

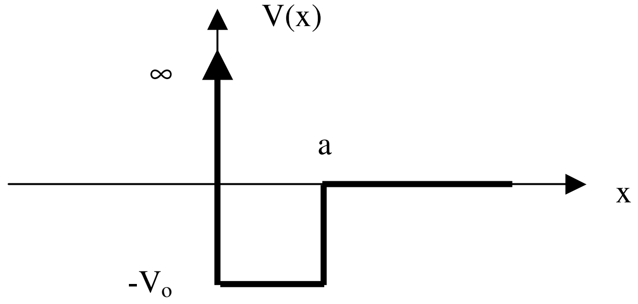



$$\phi_1(x)=Ae^{ik_1x}+Be^{-ik_1x} \qquad \phi_2(x)=Ce^{ik_2x}$$
$$A+B=C \qquad k_1(A-B)=k_2C$$

16. * Solve the equations in the preceding problem and calculate the transmission and reflection coefficients.

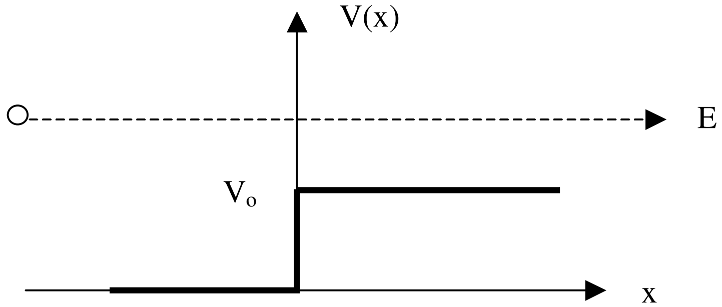
$$A=\left(\frac{\alpha+1}{2}\right)C \qquad B=\left(\frac{1-\alpha}{2}\right)C \qquad \alpha=\frac{k_2}{k_1}$$
$$R=\left(\frac{1-\alpha}{1+\alpha}\right)^2 \qquad T=1-R$$

17. Consider the potential shown below. Sketch the form of the lowest few wavefunctions. What is the connection between the solutions of this problem and those of the rectangular square well ? Consider both bound and unbound states.




$$V(x)=\begin{cases} \infty & x<0 \\ -V_o & 0<x<a \\ 0 & x>a \end{cases}$$

18. A particle of energy E > $V_o$ is incident from the left onto a potential step, as shown. For what value of the energy will the reflection and transmission coefficients be equal ?




$$\left[\frac{E}{V_o}=1.03\right]$$

19. * Solve the equations for the matching conditions for the rectangular potential barrier problem and show that the transmission coefficient is given by $T=\left[1+\frac{V_o^2}{4E(V_o-E)}\sinh^2(k_2 a)\right]^{-1}$ where the symbols have their usual meanings.

20. A positron is incident on a rectangular barrier of height 10 eV and width 1.0 Å. What is the probability that the particle will tunnel through the barrier if its energy is 1 eV, 5 eV ? [0.068, 0.332]

21. Show that the operator $\hat{H}$ defined by $\hat{H}\,\Psi(x)=\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\right)\Psi(x)$ is a linear operator for any choice of $V(x)$.

22. Show that $\phi(x) = e^{-\alpha x^2}$ is an eigenfunction of the operator $\hat{A} = -\frac{d^2}{dx^2} + x^2$ for a suitable choice of the parameter α. Find this value of α and the corresponding eigenvalue. [1/2 , 1]

23. Show that the eigenfunctions of the momentum operator have the form $e^{ipx/\hbar}$ and that the eigenvalues can be any real number *p*.

24. Consider a particle in the ground state of the infinite potential well, with wavefunction

$$\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right) \qquad 0 < x < a$$

(i) Calculate the expectation values $<x>$ and $<p>$ and comment on the significance of this result [a/2, 0]

(ii) Calculate $<x^2>$ , $<p^2>$ and the uncertainties $\Delta x$ and $\Delta p$ N.B. the uncertainty is defined as e.g. $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ $\left[0.2827a^2, \left(\frac{\hbar\pi}{a}\right)^2, 0.1808a, \frac{\hbar\pi}{a}\right]$

(iii) Evaluate the product $\Delta x \Delta p$ and comment on the result $[0.568\hbar]$

25. * Repeat problem 24 for the general state $\psi_n(x)$ for the infinite potential well. $\left[\frac{a}{2}, 0, \left(\frac{1}{3} - \frac{1}{2n^2\pi^2}\right)a^2, \left(\frac{n\hbar\pi}{a}\right)^2, 1.67\hbar\ (n=2),\ 2.63\hbar\ (n=3)\ etc\right]$

26. Verify that the functions

$$\psi_0(x) = A_0 e^{-\alpha x^2/2}$$

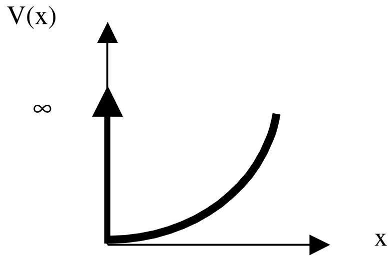
$$\psi_1(x) = A_1 x e^{-\alpha x^2/2} \qquad \alpha = m\omega/\hbar$$

satisfy the time-independent Schrödinger equation for the harmonic oscillator and obtain the normalization constants $A_0$ and $A_1$. [ $A_o = \left(\frac{\alpha}{\pi}\right)^{1/4}$ ; $A_1 = \left(\frac{4\alpha^3}{\pi}\right)^{1/4}$ ].

27. Calculate the expectation values of the kinetic and potential energies in the first excited state of the harmonic oscillator and compare your values with the relevant eigenvalues from Q.26 [ $\langle K \rangle = \langle V \rangle = \frac{3}{4}\hbar\omega$; $\langle K \rangle + \langle V \rangle = \frac{3}{2}\hbar\omega = E_1$ ]

28. Obtain the allowed energies for the potential $V(x)=\begin{cases}\infty & x<0\\ \frac{1}{2}m\omega^2x^2 & x\geq 0\end{cases}$.

(Hint: Relate this problem to the harmonic oscillator).




[$E_n=\frac{n}{2}\hbar\omega; \qquad n=3,7,11,\ldots$]

29. Show that the function $\psi(r)=e^{-r/a_0}$ satisfies the time-independent Schrödinger equation for the Hydrogen atom and find the corresponding energy.

[$E=-\frac{\hbar^2}{2ma_0^2}=-\frac{me^4}{2\hbar^2(4\pi\varepsilon_o)^2}=-13.6eV$].

30. Calculate the expectation value of the potential energy for the 1s state of the Hydrogen atom and compare this with the total energy [$\langle V\rangle=-\frac{e^2}{4\pi\varepsilon_o a_0}=-27.2eV=-2E$].

31. Look up the expression for the radial part of the eigenfunction of an electron in the 2p state of Hydrogen. Obtain an expression for the probability density for this state and determine the value of r (in terms of the Bohr radius) at which the electron is most likely to be 'found'. [r = 4 $a_0$]

32. Calculate the wavelengths of the 'longest' two lines in the Lyman series in the Hydrogen spectrum. [1.219 x $10^{-7}$ m and 1.028 x $10^{-7}$ m].

33. Show that in the limit of large *n*, the frequency of radiation emitted when a Hydrogen atom makes a transition (n+1) → n is given by $f=\frac{27.2e}{hn^3}$.

34. Show that $\frac{\partial}{\partial\phi}=-y\frac{\partial}{\partial x}+x\frac{\partial}{\partial y}$ and hence show that $\hat{L}_z=-i\hbar\frac{\partial}{\partial\phi}$. Hence show that $\hat{L}_z\psi_{nlm}=m\hbar\psi_{nlm}$

35. A one-electron atom is in a quantum state $^2F_{5/2}$. What are the values of the spin, orbital and total angular momentum quantum numbers ? Calculate the angles between the three angular momentum vectors **L**, **S** and **J** and illustrate with a diagram. [S=1/2, L=3, J=5/2, ∠**L**,**J** = 12.56°, ∠**L**,**S** = 48.19°, ∠**S**,**J** = 119.25° ]

36. For the case in problem 35, calculate the possible angles between **J** and the z axis and illustrate with a diagram. [32.31°, 59.53°, 80.27°, 99.73°, 120.47°, 147.69° ]