9. Specific heats of ideal gases

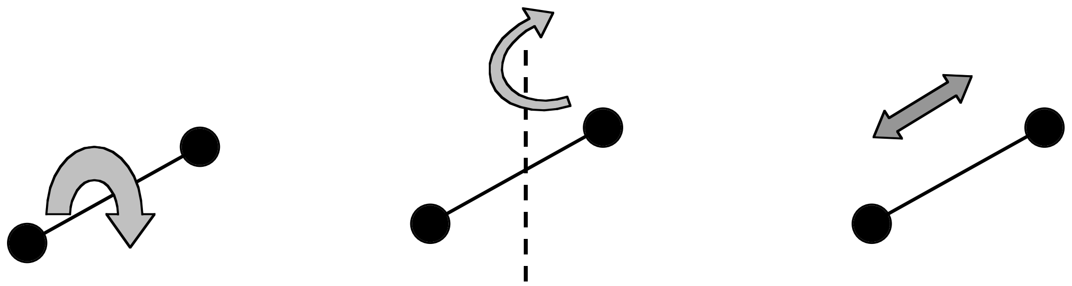
We have seen that there is a direct link between temperature and the kinetic theory of gases through the average kinetic energy per molecule:

$$\mathrm{KE}_{av} = \frac{3}{2}kT$$

We will now find expressions for the internal energy and specific heats of gases.

9.1 Internal energy of monatomic gases

We consider only monatomic gases (e.g. He, Ne, Ar) since we are concerned with the **translational** KE only. Diatomic molecules like N_2, O_2 etc. can take up energy by *rotating* or *vibrating*:



The internal energy of a monatomic gas composed of N molecules is then given by

$$U_{\text{int}} = N\left(\frac{3}{2}kT\right)$$

Again using

$$N = nN_A$$

we have

$$U_{\text{int}} = nN_A\left(\frac{3}{2}kT\right)$$

and since $k = R/N_A$ we can write

$$U_{\text{int}} = \frac{3}{2}nRT \qquad \text{internal energy of a monatomic gas}$$

Note that this depends only on T and the quantity (no. of moles) of the gas.

9.2 Specific heat of an ideal monatomic gas, constant volume

Specific heat is the quantity of heat required to raise the temperature of a given amount of substance (in this case we deal with specific heat of gases **per mole** rather than per kilogram).

The molar specific heat is defined as

$$c_m = \frac{1}{n}\frac{dQ}{dT} \qquad \textbf{molar specific heat}$$

We are interested here in the **specific heat at constant volume**:

$$c_V = \frac{1}{n}\left(\frac{dQ}{dT}\right)_V \qquad \textbf{specific heat at constant volume}$$

subscript *V* denotes
‘at constant volume’

Using the first law of thermodynamics,

$$dQ = dU + dW$$

$$\downarrow$$

$$dQ = dU + PdV$$

Since we are interested in the constant volume process $dV = 0$, i.e.

$$dQ = dU + \cancel{PdV}$$

so

$$dQ = dU$$

and,

$$c_V = \frac{1}{n}\left(\frac{dQ}{dT}\right)_V \qquad \Rightarrow \qquad c_V = \frac{1}{n}\frac{dU}{dT}$$

Having replaced Q by U we can use $U_{\text{int}} = \frac{3}{2}nRT$ (see above),

and

$$c_V = \frac{1}{n}\frac{d}{dT}\left(\frac{3}{2}nRT\right)$$

$$\boxed{c_V = \frac{3}{2}R}$$

Ideal monatomic gas, specific heat at constant volume

Since we know $R = 8.31\ JK^{-1}$ we find the value

$$c_V = \frac{3}{2}(8.31) = 12.47\ Jmol^{-1}$$

Experimentally measured values of c_V for He, Ne, Ar, Kr, Xe are all close to this value – the theory works!

9.3 Specific heat of an ideal monatomic gas, constant pressure

Again applying the first law of thermodynamics we can find c_P the specific heat at constant pressure for an ideal gas.

We find the result (see p 469 of HRW; Section 19.6 of Tipler)

$$c_P = c_V + R$$

molar specific heat at constant pressure, **any** ideal gas

c_P is just larger than c_V by an amount R

Alternatively,

$$c_P - c_V = R$$

For a monatomic ideal gas we found $c_V = \frac{3}{2}R$ so we now know that

$$c_P = \frac{3}{2}R + R = \frac{5}{2}R$$

molar specific heat at constant pressure, **monatomic** ideal gas