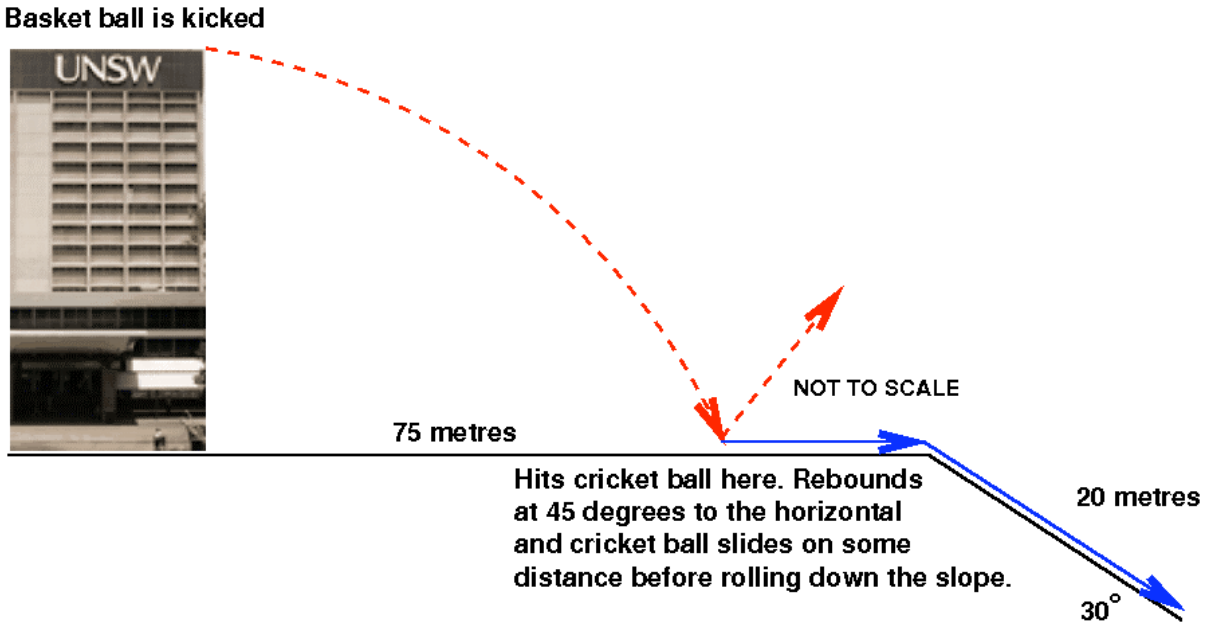
Question 1. [Marks 24]

(a) A basket ball of 600 gramme mass is kicked horizontally from the edge of a building and hits a cricket ball, located at 75 metres from the base of the building 2.5 seconds later. Stating any assumptions you use, calculate:



(i). The height of the building.

(ii). The velocity at which the basket ball hits the cricket ball.

(b) The basket ball impacts with a stationary cricket ball, of 160 gramme mass. The basket ball rebounds at an angle of 45° to the horizontal, as shown in the figure. Upon the impact, the cricket ball moves off horizontally with no vertical component. Assuming the balls do not spin and that no momentum is imparted to the Earth, calculate:

(i). The speed of the basket ball immediately after impact.

(ii). The speed of the cricket ball immediately after impact.

(iii). We have deliberately chosen balls made of stiff materials. Calculate whether or not the collision is perfectly elastic.

(c) Due to friction, the cricket ball slows down to rest by time it has slid to the edge of the slope, 10 seconds later.

(i). What is the coefficient of kinetic friction of the surface?

(ii). What is the work done by the friction in this time?

(d) The ball, which has a constant density, then rolls down the slope which has length 20 metres inclined at 30° to the horizontal. If there is no slipping, calculate its speed at the bottom of the slope.

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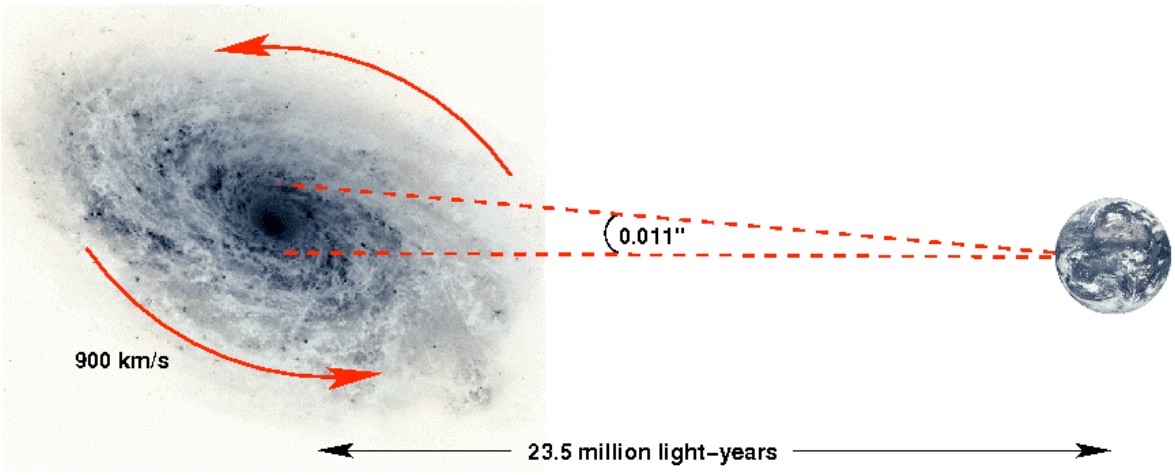
Question 2. [Marks 16]



(a) The figure (which is not to scale!) shows the rotation of the disk of material around the super-massive black hole in the centre of the galaxy NGC 4258, which is at a distance of 23.5 million light-years from the Earth (1 light-year is 9.46×10^{15} metres).

(i). Given that the disk subtends an angle of 0.011 arc-seconds (1 arc-second, 1"=1/3600 degrees) when viewed from Earth, determine the physical radius of the disk (you may need the small angle approximation, i.e. $\tan\theta\approx\theta$, where θ is in radians).

(ii). Calculate the enclosed mass, if the edge of the disk rotates at 900 km/s.

(b) Our own Galaxy, which resembles a thin disk, has a radius of about 50000 light-years (1 light-year is 9.46×10^{15} metres), which rotates once every 225 million years.
An idealized model for the Galaxy is that the disk has a uniform mass distribution and rotates as though it were a solid body.

(i). Show that the moment of inertia of a thin disk of constant density is given by

$$I=\frac{1}{2}mR^2$$ where m is the mass of the disk and R is its radius.

(ii). Calculate the rotational kinetic energy of our Galaxy in this idealized model.

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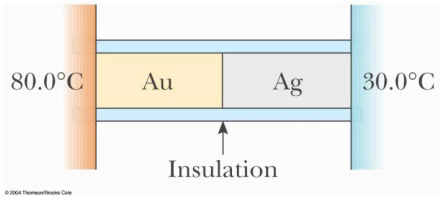
Question 3 [22 marks]

(a) A water heater is operated by solar power. If the solar collector has an area of 6.00 m^2, and the intensity delivered by sunlight is 550 W/m^2, how long does it take to increase the temperature of 1.00 m^3 of water from 20.0°C to 60.0°C?

(b) In a 30.0 s interval, 500 hailstones strike a glass window of area 0.600 m^2 at an angle of 45.0° to the window surface. Each hailstone has a mass of 5.00 g and moves with a speed of 8.00 m/s. Assuming the collisions are elastic, find the average force and pressure on the window.

(c) A bar of gold is in thermal contact with a bar of silver of the same length and area. One end of the compound bar is maintained at 80.0°C while the opposite end is at 30.0°C. When the energy transfer reaches steady state, what is the temperature at the junction?



Note: Thermal conductivities are
Gold: k_{Au} = 314 W/m.°C
Silver: k_{Ag} = 427 W/m.°C

Question 4 [14 marks]

A 2.00 kg object is attached to a spring and placed on a horizontal, smooth surface. A horizontal force of 20.0 N is required to hold the object at rest when it is pulled 0.200 m from its equilibrium position (the origin of the x axis). The object is now released from rest with an initial position of x_i = 0.200 m, and it subsequently undergoes simple harmonic oscillations.

(a) Find the force constant of the spring.
(b) Find the frequency of the oscillations.
(c) Find the maximum speed of the object. Where does this maximum speed occur?
(d) Find the maximum acceleration of the object. Where does it occur?
(e) Find the total energy of the oscillating system.
(f) Find the speed of the object when its position is equal to one third of the maximum value.
(g) Find the acceleration of the object when its position is equal to one third of the maximum value.

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Question 5 [24 marks]

(a) A violin string has a length of 0.350 m and is tuned to concert G, with f_G = 392 Hz.

(i) Where must the violinist place her finger to play concert A, with f_A = 440 Hz?

(ii) When the violinist plays concert A, her finger causes the tension in the string to increase slightly. What is the maximum allowable percentage change in string tension if the pitch is to be correct to within ± one half a finger width (i.e. to within 0.600 cm)?

(b) Two loudspeakers are placed on a wall 2.00 m apart. A listener stands 3.00 m from the wall directly in front of one of the speakers. A single oscillator is driving the speakers at a frequency of 300 Hz.

(i) What is the phase difference between the two waves when they reach the observer?

(ii) What is the frequency closest to 300 Hz to which the oscillator may be adjusted such that the observer hears minimal sound?

Question 6 [20 marks]

A car travels clockwise at constant speed around a circular track, as shown below. It emits a constant sound at a frequency of ν_0 kHz. An observer, sitting at position P, hears the pitch of this sound rise and fall as the car goes around the track.

(a) We wish to see *qualitatively* how the frequency varies, and a comparison of the maximum and minimum observed frequencies observed for the various cases.

(i) Plot a graph of frequency (plotted vertically) versus time (plotted horizontally) showing the frequency observed by a person sitting at position f as the car makes one full circuit of the track. Mark on the graph the times at which the car is at positions a, b, c and d.

(ii) On the same graph, plot the frequency observed by a person sitting on the track at position c.

(iii) On the same graph, plot the frequency observed by a person sitting at position e.

(iv) On the same graph, plot the frequency observed by a person sitting at position O (the centre of the circle).

(b) Let the radius of the track be *r*, the speed of the car be *s*, and the distance between O and P be *x*. Write down an expression for the maximum and minimum frequencies heard at position P.