PHYS1121 & PHYS1131

TEST 2 (MECHANICS) 2008 - SOLUTIONS

1. (a) The main assumption is that there is no air resistance present.
   [1 mark off for this part if this is not stated.]
   The students may also state that the acceleration due to gravity is the same at the top and bottom of the building and that the horizontal distances are perfectly flat, but to state these is not necessary in this case.
   i. $s_y = u_y t + \frac{1}{2}at^2 = 0 - \frac{1}{2}(9.8)(2.5)^2 = 30.6$ m. **[2]**
   ii. $v_x = s_x/t = 75/2.5 = 30$ m/s **[1]**, $v_y = u_y + at = 0 - 9.8(2.5) = 24.5$ m/s **[1]**
   $v = \sqrt{30^2 + 24.5^2} = 38.7$ m/s **[1]**, $\tan\theta = (24.5/30.0) \Rightarrow \theta = 39.2°$ **[1]**

   (b) i. Conservation of momentum, $m_B u_B + m_C u_C = m_B v_B + m_C v_C$
   x-component: $0.6 \times 30 + 0 = 0.6\cos 45° \times v_B + 0.16\,v_C$
   $18 = 0.424\,v_B + 0.16\,v_C$
   y-component: $0.6 \times 24.5 + 0 = 0.6\sin 45° \times v_B + 0$
   $14.7 = 0.424\,v_B \Rightarrow v_B = 34.7$ m/s. **[4]**
   ii. $0.16\,v_C = 18 - 0.424\,v_B = 18 - 14.7 \Rightarrow v_B = 20.6$ m/s **[2]**
   iii. **[1131 only]** $KE_i = KE_f$ for perfectly elastic collision
   $KE_i = \frac{1}{2}m_B\,u_B^2 = \frac{1}{2}(0.6)(38.7)^2 = 450$ J
   $KE_f = \frac{1}{2}m_B\,v_B^2 + \frac{1}{2}m_C\,v_C^2 = 0.3(34.7)^2 + 0.08(20.6)^2 = 395$ J
   i.e. Kinetic energy is lost, so collision is not perfectly elastic. **[2]**

   (c) i. $a = (v_C - u_C)/t = 20.6/10 = -2.06$ m s^{-2}. **[1]**
   $F = ma = 2.06 \times 0.16 = 0.33$ N in negative x-direction **[1]**
   Coefficient of kinetic friction, $\mu = F/N = F/mg = 0.21$ **[2]**
   ii. $v^2 = u^2 + 2as \Rightarrow s = -(20.6)^2/(2 \times -2.06) = 103$ m
   $W = F\,s = 0.33 \times 103 = 34$ J **[2]**

   (d) **[1131 only]** $PE_i + KE_i = PE_f + KE_f$
   $mgh + \frac{1}{2}m\,u^2 = \frac{1}{2}m\,v^2 + \frac{1}{2}I\,\omega^2$
   For a solid sphere, $I = \frac{2}{5}mr^2 \Rightarrow KE_f = \frac{1}{5}m\,r^2\omega^2$
   $v = r\,\omega \Rightarrow KE_f = KE_f = \frac{1}{5}m\,r^2\frac{v^2}{r^2} = \frac{1}{5}m\,v^2$
   $mgh + \frac{1}{2}m\,u^2 = \frac{1}{2}m\,v^2 + \frac{1}{5}m\,v^2$
   $gh + \frac{1}{2}u^2 = \frac{1}{2}v^2 + \frac{1}{5}v^2$
   $\Rightarrow \frac{7}{10}v^2 = gh + \frac{1}{2}u^2$
   $h = 20\sin 30° = 10$ m, $g = 9.80$ m s^{-2} and $u = 0$,
   so $v^2 = \frac{10}{7}(9.8)(10) \Rightarrow v = \sqrt{140} = 11.8$ m/s **[4]**

[Total 24]

(a) i. Half angle is $0.011/2 = 0.0055''$ so $(0.0055/3600) \times (\pi)(180) = 2.67 \times 10^{-8}$ radians, $\tan\theta \approx \theta \Rightarrow r \approx \theta \times D = 2.67 \times 10^{-8} \times 23.5 \times 10^6 = 0.63$ light-years.

1131 - 1 light-year $= 3 \times 10^8 \times 3600 \times 24 \times 365 = 9.46 \times 10^{15}$ metres

The students can then convert this to $0.63 \times 9.46 \times 10^{15} = 6.0 \times 10^{15}$ metres, if they like. It is also acceptable to have converted the 23.5×10^6 light-years to metres first (or leave the answer in light-years).
Also, the students, may not have used the half angle, giving the *diameter* as $0.011'' = 5.33 \times 10^{-8}$ radians $\Rightarrow d = 1.25$ light-years. This is also acceptable, provided they then halve this to give the radius. If they don't, dock one mark **[4]**.

ii. By equating the centripetal and gravitational forces $\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow M = v^2 r/G = (900 \times 10^3)^2 \times 6.0 \times 10^{15}/6.67 \times 10^{-11} = 7.2 \times 10^{37}$ kg, which they can convert to 3.6×10^7 solar masses **[4]**

(b) i. **[1131 only]** Moment of inertia, $I \equiv \int r^2\, dm$, and since $dm = \rho\, dV$
$I = \int r^2 \rho\, dV$.
For a thin disk, $dV = 2\pi\, r\, dr\, L$, where L is the height of the disk
$\Rightarrow I = \int r^2\, \rho\, 2\pi\, r\, L\, dr$.
Taking the constants out of the integral and integrating between 0 and R,
$\Rightarrow I = 2\pi\, \rho\, L \int_0^R r^3\, dr = 2\pi\, \rho\, L\, [r^4/4]_0^R = \frac{1}{2}\pi\, \rho\, L\, R^4$
For a constant density, $\rho = m/V = m/(\pi\, R^2\, L) \Rightarrow I = \frac{1}{2} m\, R^2$ **[4]**

ii. The work required is equivalent to the rotational kinetic energy, which is $KE = \frac{1}{2} I\, \omega^2$, which given $I = \frac{1}{2} mR^2$, becomes $KE = \frac{1}{4} m\, v^2$.
$v = 2\pi\, R/T$ and so for a radius of $50\,000 \times 9.46 \times 10^{15} = 4.7 \times 10^{20}$ metres and a period of $225 \times 10^6 \times 3600 \times 24 \times 365 = 7.1 \times 10^{15}$ seconds, $v = 420 \times 10^3$ m/s.
Again, equating the centripetal and gravitational forces, $\frac{mv^2}{R} = \frac{GMm}{R^2}$
$\Rightarrow M = v^2 R/G = (420 \times 10^3)^2 \times 4.7 \times 10^{20}/6.67 \times 10^{-11} = 1.2 \times 10^{42}$ kg (or 6×10^{11} solar masses).
Putting this back into the above equation,
$KE = \frac{1}{4}(1.2 \times 10^{42})(420 \times 10^3)^2 = 5 \times 10^{52}$ J. **[4]**
Obviously with the repeated appearances of various terms, there's plenty of opportunity for cancelling, so the derivation of the energy without the intermediate terms is also acceptable (although the cancellations must be shown), for example $KE = \frac{1}{4} G\, m^2/r$, $KE = \frac{1}{4} v^4\, r/G$, $KE = \frac{1}{4}\left(\frac{2\pi\, r}{T}\right)^4 \frac{r}{G}$.

[Total 16]